Nordic-Baltic Physics Olympiad 2021

1. Photon rocket (5 points) — *Jaan Kalda, Oskar Vallhagen.* Consider a hypothetical interstellar travel with a photon-propelled spaceship of initial rest mass $M = 1\times10^5$ kg. The on-board fuel (antimatter) is annihilated with an equal mass of matter to create photons yielding a reactive force. The matter required for annihilation is collected from the very sparse plasma of the interstellar space (assume that the velocity of the interstellar plasma is zero in the Earth's frame of reference). The speed of light is $c = 3\times10^8$ m/s.

i) *(1 point)* What should be the initial rate μ (kg/s) at which the antimatter should be burned for the acceleration to be equal to the free fall acceleration on Earth ($g = 9.81\,\mathrm{m/s^2}$)?

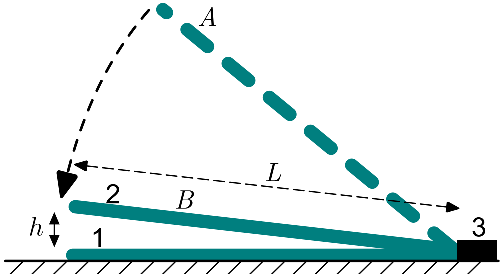
ii) *(3 points)* The engines of the spaceship are switched off when its rest mass has decreased down to $m_f = M/10$; what is its final speed?

iii) *(1 point)* The frequency of the emitted photons is measured by an observer on Earth. What is the frequency of the last photons (emitted just before the engine is switched off), as measured on Earth, if the frequency in the the space ship frame remains constant and equal to f_0?

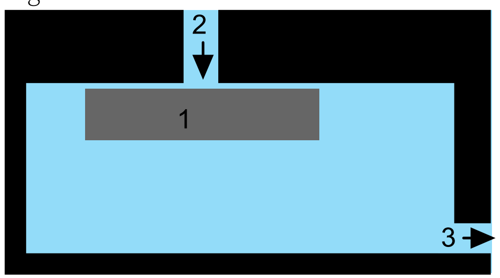
2. Gas and fluid flows (10 points) — *Jaan Kalda, Päivo Simson.*

i) *(1 point)* If you let a glass plate fall onto another glass plate, it will not break, but stop softly. The figure depicts, a plate (marked with '1') is resting on the floor, and another plate (marked with '2') is falling while a bump on the ground (marked with '3') prevents it from sliding. The falling plate started from the position A and is now at the position B, at a very small distance $h = h_0$ from the resting plate, and falling currently with the angular speed ω_0. What is the speed of air between the plates near the leftmost edge?

ii) *(2.5 points)* The glass plate has width $L \gg h_0$, thickness $t \ll L$, density ρ_g, and its length (into the depth of the figure) is much bigger than L. How will the angular speed of the plate depend on h during its subsequent motion if the density of air is ρ_a? Neglect the gravity as well as the viscosity and compressibility of the air. Assume that the air flow remains everywhere laminar.



iii) *(3 points)* A cylindrical stone disc (marked with '1' in the figure) of radius R, thickness h and density ρ_s is pressed against the ceiling of a basin filled with water of density ρ_w. Small bumps on the surface of the ceiling maintain a small gap of thickness $t \ll R$ between the ceiling and the surface of the disk. Water flows from a pipe (marked with '2'; the outflow pipe '3' is far away) of radius $r \ll R$ coaxially with the disk into the basin, see the figure. The radius of the pipe is much bigger than the gap between the disk and the ceiling, i.e. $r \gg t$. What should be the mass flow rate μ (kg/s) from the pipe so as to keep the disk from falling down? The free fall acceleration is g.



iv) *(0.5 points)* Steam turbines are widely used in power plants. According to a simplified model, water is being boiled at temperature $t_t = 180\,^\circ$C and pressure $p_t = 1\times10^6$ Pa (real steam turbines can have much higher pressures than that), and the created vapour flows out through a cylindrical channel of cross-sectional area $A = 1\,\mathrm{cm^2}$ in the wall; the ambient pressure $p_0 = 1\times10^5$ Pa. Find the entropy difference ΔS of one mole of vapour and one mole of liquid water (molar mass $M = 18$ g/mol, latent heat of vaporisation at $100\,^\circ$C: $L = 2.3$ MJ/kg) in the outflowing jet.

v) *(3 points)* Find the mass flow rate μ of the created steam jet, as well as the relative mass content r of liquid-phase-water in it. Assume that while flowing into and in the channel, the expansion of water vapours is reversible (i.e. heat conductivity can be neglected, and that there is always equilibrium between the liquid and gaseous phases); the adiabatic index of water vapours $\gamma = 4/3$.

3. Rotating space station (13 points) — *Jaan Kalda, Kaarel Hänni.* A space station at a geostationary orbit has a form of a cylinder of length $L = 100$ km and radius $R = 1$ km is filled with air (molar mass $M = 29$ g/mol) at the atmospheric pressure and temperature $T = 295$ K and the cylindrical walls serve as a ground for the people living inside. It rotates around its axis so as to create normal gravity $g = 9.81\,\mathrm{m/s^2}$ at the "ground".

i) *(0.5 points)* What is the rotation period τ?

ii) *(2 points)* A ball is thrown from a certain point on the "ground", and caught $t = \tau/2$ later at the very same point. What was the throwing speed of the ball? Neglect air drag.

iii) *(2 points)* A spherical balloon of radius $r = 3$ m is filled with helium (molar mass $M' = 4$ g/mol), and is used to lift a weight of unknown mass m. The weight is fixed to the ball with a light rope of length $L = 20$ m, and the system rises until coming to a stop at the height $H = 500$ m from the "ground". Determine the value of the mass m.

A rope of linear mass density $\lambda = 1$ kg/m is fixed to the "ground" at two diametrically opposite points of the cylinder (so that the distance between the endpoints of the rope is $2R$). Let A, B, and C denote the two endpoints and the middle point of the rope, respectively.

iv) *(1.5 points)* Assuming that the height of the point C above the "ground" is h, determine $T_A - T_C$, the difference of the tension forces in the rope at the points A and C.

v) *(1.5 points)* Assuming that at the point A, the rope meets the "ground" at an angle α, determine the ratio of the tension forces T_A/T_C.

vi) *(1.5 points)* Find T_C if $h = 495$ m by approximating the shape of the rope with a parabola.

vii) *(2 points)* The metallic walls of the space station carry a total charge Q. Inside the space station, a charged ball hovers above the "ground" motionlessly. Find the charge-to-mass ratio q/m of the ball. Ignore the effect of charges induced by the charged ball on the "ground".

viii) *(2 points)* Gauss theorem states that $\oint \vec{E}\cdot \mathrm{d}\vec{A} = 0$, where integral is taken over a closed surface embracing a volume V with no charges inside. How should this equality be modified by an observer on board of the space station if there are no other charges than the total charge Q distributed over the perimeter of the station?

4. Stretching gloves (8 points) — *Eero Uustalu.*

Tools At least three pairs of large colorless medical semi-transparent latex rubber gloves; a roll of transparent and strong office tape; a pair of sharp scissors; at least four sheets of A4 or larger size graphing paper; three rulers; a flexible measuring tape with a length of at least one meter; extra-fine point universal surface marker. Rubber gloves can be cut as needed into pieces. The pieces of gloves can be fixed to your working table either directly using the tape and/or with the help of a ruler (to achieve a firmer fixing).

Latex is a highly stretchable elastic material for which it can be assumed that its volume remains constant during stretching, up to the breaking point.

For each of the tasks, sketch your experimental setup and explain the steps you made to obtain the best possible precision, and tabulate the directly measured data.

i) *(1 point)* Determine the maximal strain ε_m of the latex film band (i.e. the strain by which the band breaks). Strain is defined as the relative change in length, $\varepsilon = (l - l_0)/l_0$, where l and l_0 are the stretched and unstretched lengths of the band.

ii) *(7 points)* Determine and plot the stress-strain relationship for latex bands. Stress is defined as the tension force per cross-sectional area. Express the stress σ in relative units, normalised to the maximal stress at the breaking point.