1. Phases of water (6 points) — *Johan Runeson.*

In this problem we will look at the phase diagram of water (see graphs on a separate page). The first figure shows the phase diagram in a region close to the triple point [(s) – solid, (l) – liquid, (g) – gas], while the second figure shows the melting curve. When two phases α and β are in equilibrium, the phase transition curve follows the law of Clausius–Clapeyron:

$$\frac{\mathrm{d}p}{\mathrm{d}T} = \frac{1}{T}\frac{H_\beta - H_\alpha}{V_\beta - V_\alpha},$$

where H_α is the specific enthalpy (enthalpy per mass) of phase α, and V_α is the specific volume (volume per mass).

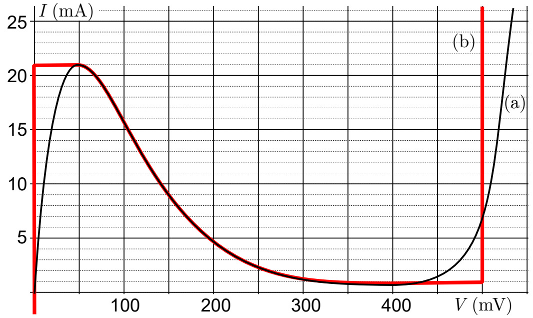
i) *(1.5 points)* Using that $V_g \gg V_l$, find an expression for the *liquid-gas* transition curve $p(T)$ in terms of the latent heat of evaporation $\Delta H_{lg} \equiv H_l - H_g$, the pressure p_0 at any reference point along the curve, the gas constant R and the molar mass μ.

ii) *(1.5 points)* Approximate the Earth as a system of a homogeneous atmosphere, consisting of air and water vapor, in equilibrium with a sea of liquid water. If the atmospheric temperature rises by $3\,^\circ\mathrm{C}$, by what percentage does the water vapor pressure rise? (The current temperature of the Earth is $15\,^\circ\mathrm{C}$.) You may need the values $R = 8.314\,\mathrm{J\,mol^{-1}\,K^{-1}}$ and $\mu = 18.015\,\mathrm{g\,mol^{-1}}$.

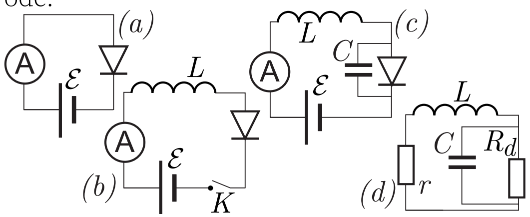
iii) *(3 points)* Use reasonable approximations and calculate $V_l - V_s$, the difference in specific volume between liquid water and ice, at atmospheric pressure.

2. Tunnel diode (10 points) — *Jaan Kalda.*

The V-I-curve of a tunnel diode is depicted in the figure below, curve (a). In some parts of the problem, we use an idealized model curve (b).



i) *(1 point)* In order to measure the V-I curve of the diode, it is connected in series with a variable power supply (the value of the electromotive force $\mathcal{E}$ can be changed from $0\,\mathrm{V}$ to $1\,\mathrm{V}$), see circuit (a). The ammeter has internal resistance $r = 2\,\Omega$; the applied voltage is $\mathcal{E} = 50\,\mathrm{mV}$. What is the diode voltage V_i and current I_i? Use the real V-I-curve of the diode.



ii) *(1 point)* Now, let us study the effect of the self-inductance of the wires. In order to take into account such an inductance, the circuit needs to be modified as shown in circuit (b); let $L = 500\,\mathrm{nH}$. The switch K is kept open until the voltage is adjusted to $\mathcal{E} = 250\,\mathrm{mV}$, and is then closed. How long will it take for the current to reach $I_1 = 20\,\mathrm{mA}$? Neglect henceforth (until otherwise instructed) the internal resistances of the battery and of the ammeter (put $r = 0$), and use the idealized V-I-curve of the diode.

iii) *(1 point)* With the same settings as for task ii), how long will it take from the moment when the switch was closed until the diode voltage reaches $V_2 = 500\,\mathrm{mV}$?

iv) *(2 points)* With the same setting as for task ii), plot the diode current as a function of time and find the period and amplitude of the current oscillations.

v) *(2 points)* Circuit (b) is used to measure the V-I curve of the diode: for each data point, the voltage is adjusted to the desired value while the switch is kept open, and then the switch is closed. Note that when the ammeter current oscillates with a high frequency, it shows **the average current**. Plot the expected measurement results, i.e. the average current through the ammeter as a function of the applied voltage $V = \mathcal{E}$.

vi) *(1 point)* Thus far we have assumed that the diode is an ideal device; in reality, it has a small parasitic capacitance, let it be $C = 30\,\mathrm{pF}$. Taking this into account, our circuit should be drawn as shown in diagram (c). Now we assume the ammeter, again, to be non-ideal, with internal resistance $r = 2\,\Omega$. Let us assume that after closing the switch, the voltage was slowly increased from $\mathcal{E} = 0\,\mathrm{mV}$ to $\mathcal{E} = 150\,\mathrm{mV}$ so that a stationary (oscillations-free) operation regime $V(t) \equiv V_0$ and $I(t) \equiv I_0$ has been achieved. Suppose there is a small perturbation to the diode current and voltage: $I = I_0 + \delta I(t)$ and $V = V_0 + \delta V(t)$, where I_0 and V_0 are the current and voltage in the stationary operational regime. For small perturbation amplitudes, the V-I-curve of the diode can be linearized, resulting in $\delta V = R_d \delta I$, where R_d is the *differential resistance* of the diode. Determine the value of R_d.

vii) *(2 points)* Continuing with the previous question, one can show that the problem of stability for the circuit (c), i.e. the question if the small current perturbations $\delta I(t)$ will grow exponentially in time or not, is equivalent to the problem of stability for the circuit (d) (the battery is removed, and the diode is substituted with its differential resistance found by the previous task). What is the largest inductance of wires L for which the system is stable?

3. Conical room (3 points) — *Maté Vigh.*

The interior of a modern museum is a perfect, right cone with half apex angle $60°$ (i.e. the walls are slanted by $60°$ with respect to the vertical). Launched from the center of the cone's base, the minimal speed needed for a projectile to reach the apex (i.e. the highest point of the walls) is v_0. What is the minimal speed required to reach the wall of the cone?

4. Drone (9 points) — *Lasse Franti and Jaan Kalda.*

A drone is pulling a cuboid with a rope as shown in the sketch; the cuboid is sliding slowly, with a constant speed, on the horizontal floor. The cuboid is made from an homogeneous material. You may take measurements from the sketch (on a separate page) assuming that the dimensions and distances on it are correct within an unknown scale factor. In order to help you in case you don't have access to a printer, and need to read the problem texts directly from the computer screen, some auxiliary dashed lines are shown in the diagram (which might or might not be useful).

i) *(2 points)* Find the coefficient of friction between the cuboid and the floor.

ii) *(2 points)* Find the mass of the cuboid if the mass of the drone is $m = 1\,\mathrm{kg}$.

iii) *(2 points)* Next we will be studying the flight of a drone in adiabatic atmosphere. In adiabatic atmosphere, air parcels are moving continuously up or down, and while doing so, expand or contract adiabatically. It can be shown that in adiabatic atmosphere, the temperature is a linear function of the height z: $T = T_0 - zg/c_p$, where $T_0 = 293\,\mathrm{K}$ is the temperature on the ground, $c_p = 1.00\,\mathrm{J\,g^{-1}\,K^{-1}}$ is the specific heat of air at constant pressure, and $g = 9.81\,\mathrm{m/s^2}$. Find the dependence of air density ρ as a function of height, in terms of the density ρ_0 at the ground level ($z = 0$), specific heat of air at constant volume $c_v = 0.718\,\mathrm{J\,g^{-1}\,K^{-1}}$, and other previously defined quantities.

iv) *(3 points)* Assuming that the maximal flight height $z_{\max}$ of a drone with no load is limited by the power of its motor, find $z_{\max}$ if it is known that the power is just enough for the drone to lift a load equal to 50% of its mass off the ground. You may neglect the effect of turbulence on drone's thrust.

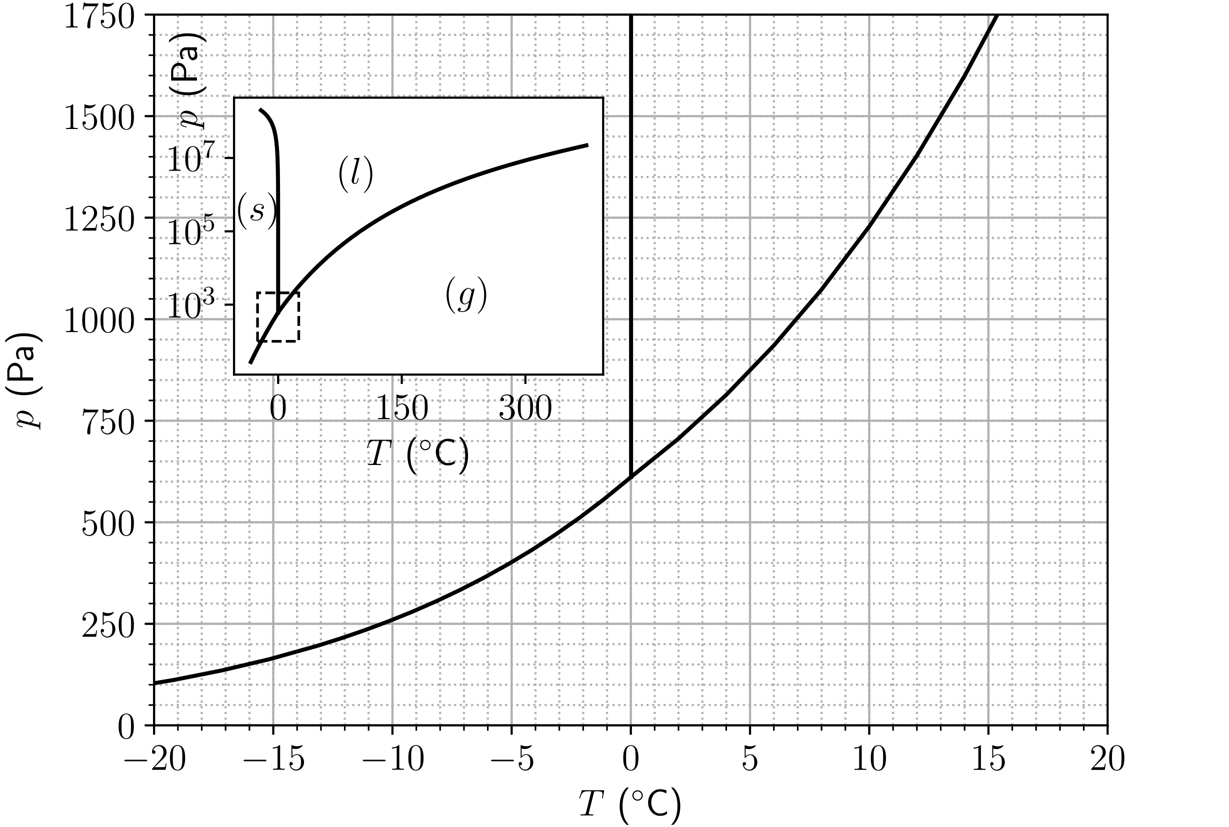
5. Bottle's sound (8 points) — *Jaan Kalda and Eero Uustalu. Tools:* an empty 1-litre bottle, a small (about 100 ml) cup of known volume (or other tool for measuring water volumes), a smartphone with installed "Physics Toolbox Sensor Suite" or "Physics Toolbox Pro" (mark on your paper, which version did you use).

If you blow near the bottle's mouth, a whistling sound can be generated: a gentle (to moderately strong) air flow needs to pass the bottle's mouth perpendicularly to the bottle's axis. Your task is to study the dependence of the frequency f of the generated sound as a function of the volume V occupied by water inside the bottle.

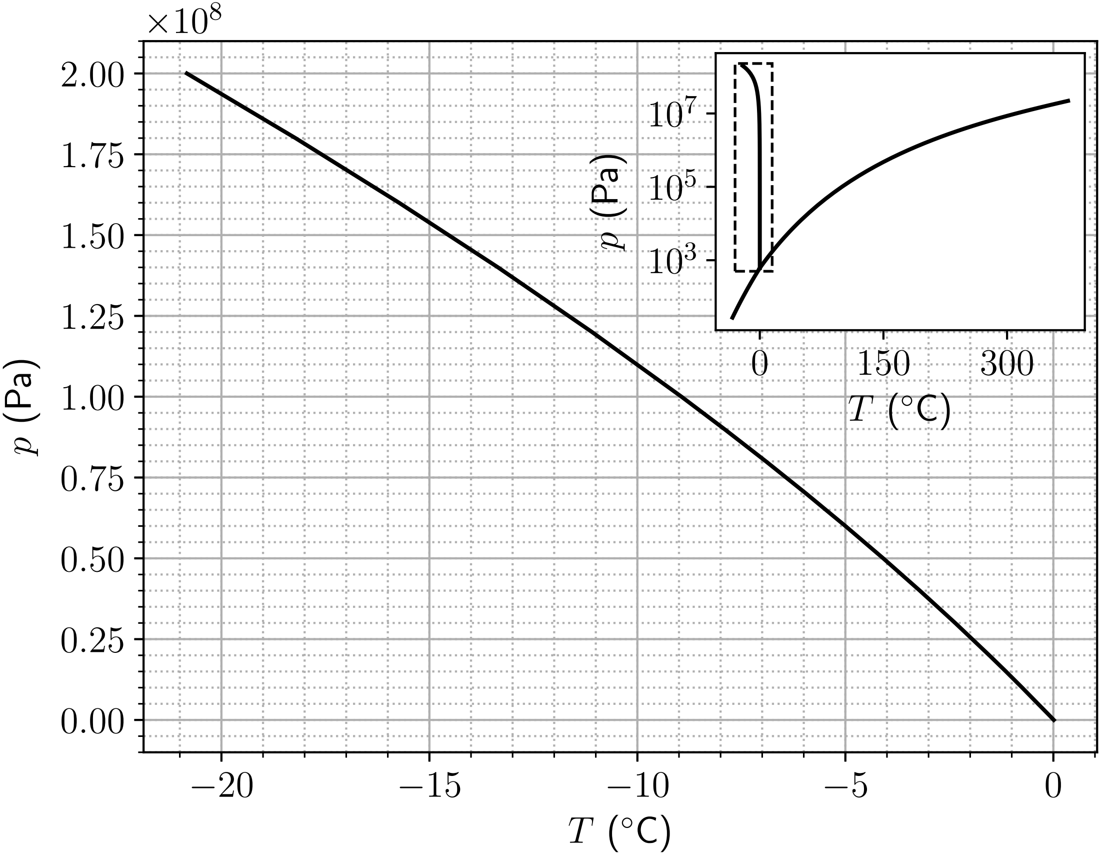
i) *(4 points)* While blowing near the bottle's mouth, measure the sound frequency using either the "tone detector" or "spectrum analyzer" of the "Physics Toolbox" (once the app is launched, the menu can be pulled from the left top corner of the screen). If you manage to obtain a nice distinct sound, use the "tone detector"; otherwise use the "spectrum analyzer" to determine the frequency of the spectrum's peak. Tabulate your measurement data.

ii) *(1 point)* Either based on theoretical consideration or on the data analysis, suggest a functional dependence of f on V.

iii) *(3 points)* Test the validity of your suggestion for this dependence graphically, and determine the parameters of it. Error analysis is not required.



1750
1500
1250
1000
750
500
250
0
−20
−15
−10
−5
0
5
10
15
20
T (°C)
p (Pa)
p (Pa)
10^7
10^5
10^3
(l)
(s)
(g)
0
150
300
T (°C)



$\times 10^8$
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
p (Pa)
−20
−15
−10
−5
0
T (°C)
10^7
10^5
10^3
p (Pa)
0
150
300
T (°C)