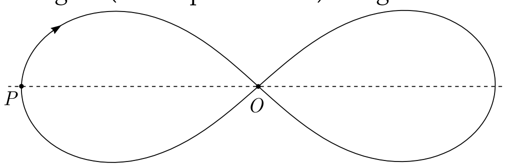
Nordic-Baltic Physics Olympiad 2018

1. GRAVITATIONAL RACING (11 points) — *Maté Vigh and Jaan Kalda.* While in generic case, the dynamics of three gravitationally interacting bodies is complicated and chaotic, there are special cases when the dynamics is regular. In particular, there are cases when bodies perform periodic motion. The simplest case of such periodic motion is when all three bodies, positioned at the vertices of an equilateral triangle, rotate as a rigid body. In what follows we consider a more complicated periodic motion.

Relatively recently [1], it was discovered that three equal point masses can move periodically along a common 8-shaped trajectory shown in the figure (arrow denotes the direction of motion). This figure is based on a computer simulation and has a correct shape. If needed, you can measure distances from the enlarged version of the figure (on a separate sheet) using a ruler.



Let us enumerate the three bodies with numbers 1, 2, and 3, according to the order in which they pass the leftmost point P shown in the figure. Let O_2 and O_3 denote the positions of the bodies 2 and 3, respectively at that moment when the body 1 is passing the middle point O. Similarly, let P_2 and P_3 denote the positions of the bodies 2 and 3, respectively at that moment when the body 1 is passing the leftmost point P. Let T denote the full period of motion of each of the bodies along this 8-shaped trajectory.

i) *(2 points)* Express the following travel times for one of the bodies: (a) from O_2 to O; (b) from O_3 to P_2.

ii) *(1 point)* Let $\vec{v}_1$, $\vec{v}_2$, and $\vec{v}_3$ denote the velocities of the three bodies at a certain moment of time. Write down an equality relating these three vectors to each other.

iii) *(2 points)* Prove that the total angular momentum of this system is zero.

iv) *(2 points)* Construct the positions of the points O_2 and O_3 (use the figure on the separate sheet). Motivate your construction.

v) *(2 points)* Construct the positions of the points P_2 and P_3 (use the figure on the separate sheet). Find two independent ways of construction and motivate your methods.

vi) *(2 points)* Find the ratio of the speed of the bodies at point O to the speed at point P.

2. SPEED CAMERA (6 points) — *Mihkel Kree.* In this problem, we analyse the working principle of a speed camera. The transmitter of the speed camera emits an electomagnetic wave of frequency $f_0 = 24\,\mathrm{GHz}$ having waveform $\cos(2\pi f_0 t)$. The wave gets reflected from an approaching car moving with speed v. The reflected wave is recorded by the receiver of the speed camera.

i) *(2 points)* Express the freqency f_1 of the reflected wave.

ii) *(2 points)* In the speed camera, the received waveform is multiplied with the original emitted waveform. Express all the frequency components present in the multiplied signal.

iii) *(2 points)* Given the lowest frequency component $f_{\mathrm{low}} = 4.8\,\mathrm{kHz}$ present in the multiplied signal, calculate the speed of the car v. *Note:* the speed of light $c = 3.0 \times 10^8\,\mathrm{m/s}$ and the following trigonometric identity might turn to be useful:

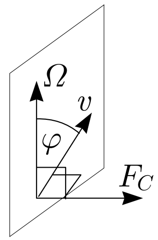
$$\cos\alpha\cos\beta = \tfrac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right].$$

3. WEATHER FORECAST (7 points) — *Johan Runeson.* The map shown on a separate page shows isobars at a constant height close to sea level. You may assume that the pattern of isobars is stationary in time (changes very slowly).

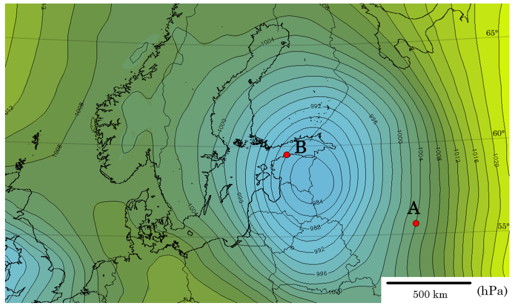
i) *(2 points)* Sketch the direction of the wind velocity at the points A and B.

ii) *(2.5 points)* Estimate the magnitude of the wind velocity at point A. Make use of the fact that at point A, the lines of constant pressure are almost straight. The density of air near the ground $\rho = 1.23\,\mathrm{kg/m^3}$.

iii) *(2.5 points)* Estimate the magnitude of the wind velocity at point B.

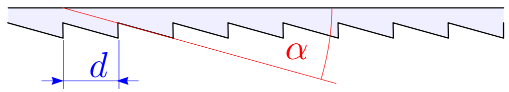


Hint. When an object of mass m (for example a slab of air) is moving with velocity v in a frame of reference rotating with angular velocity Ω, it feels a fictitious force called the *Coriolis force* given by $F_C/m = 2v\Omega\sin\varphi$, where the angle φ and the directions are indicated in the figure below.



4. FRESNEL PRISM (12 points) — *Eero Uustalu and Jaan Kalda.* **Equipment:** a sheet of Fresnel prism, a sheet with purple and magenta stripes (see separate sheet), a piece of cardboard paper (can be used as a screen), ruler, measuring tape, stand, green laser ($\lambda_0 = 532\,\mathrm{nm}$). NB! Avoid looking into direct or reflected laser light, it my damage your eyes! The intensity maximum for the scattered light from magenta stripes is $\lambda_m = 630\,\mathrm{nm}$, and from cyan stripes — $\lambda_c = 495\,\mathrm{nm}$.

Fresnel prism is a transparent sheet with a periodic array of stripes; cross-section of such a sheet is shown in figure. The refraction index of the material from which the sheet is made $n = 1.47$.



i) *(4 points)* Determine the pitch d of the Fresnel prism (see figure for the definition of the pitch).

ii) *(4 points)* Determine the angle α of the prism.

iii) *(4 points)* Assuming that in the range of visible light, the refraction index $n = n(\lambda)$ of the Fresnel prism material is a linear function of wavelength λ, determine the chromatic dispersion $\frac{\mathrm{d}n}{\mathrm{d}\lambda}$.

---

[1] Cristopher Moore, Phys. Rev. Lett. 70, 3675 (1993)

5. MAGNETIC BILLIARD (9 points) — *Jaan Kalda.* Consider two absolutely elastic dielectric balls of radius r and mass m one of which carries isotropically distributed charge $-q$, and the other — $+q$. There is so strong homogeneous magnetic field B, parallel to the axis z, that electrostatic interaction of the two charges can be neglected; neglect also gravity and friction forces. The first ball (negatively charged) moves with speed v and collides with the second ball which had been resting at the origin. The collision is central, and immediately before the impact, the velocity of the first ball was parallel to the x-axis.

i) *(1 point)* What is the speed of the second ball immediately after the collision?

ii) *(2 points)* Sketch the trajectories of the centres of the both balls during the subsequent motion.

iii) *(3 points)* What is the average velocity (magnitude and direction) of the balls during their subsequent motion?

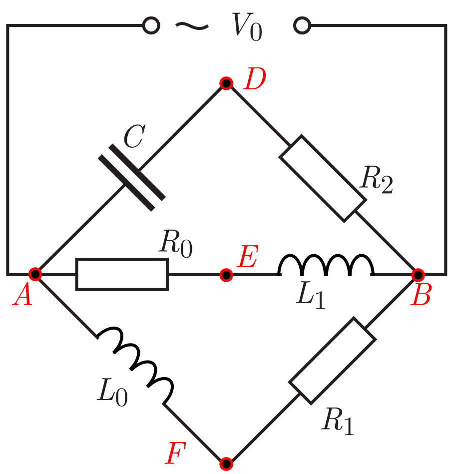
iv) *(3 points)* Consider the same situation as before, except that the there are three differences in the assumptions: the both balls have now identical positive charge $+q$; electrostatic repulsion between the balls is no longer negligible; the collision is not necessarily central (but the balls move still at the same value of z so that the collision will not induce any motion in the direction of the z-axis). Let P_i denote the point where the surfaces of the two balls are in contact during the i-th collision. What is the maximal distance between P_i and P_j (maximize over all the values $i,j = 1,\ldots\infty$, and over all the impact parameters of the collisions for fixed values of B, m, and q)?

6. CUBE (5 points) — *Taavet Kalda.* A laser pointer of power P is directed at a glass cube. The refractive index of the cube is n. The surface of the cube has anti-reflective coating so that there is no *partial* reflection when light travels from one medium to the other. The speed of light is c.

i) *(3 points)* What is the maximum force by which the laser pointer can push the cube if the laser needs to be kept parallel to one of the faces of the cube (this means that the laser beam can propagate only in a two-dimensional plane)?

ii) *(2 points)* What is the maximum force by which the laser pointer can push the cube if the orientation of the laser can be arbitrary?

7. LCR-CIRCUIT (5 points) — *Jaan Kalda.* Consider the circuit shown in the figure.



i) *(2 points)* Draw a phasor diagram for this circuit showing the voltage vectors between the following nodes: V_{AD}, V_{DB}, V_{AB}, V_{AE}, V_{EB}, V_{AF}, and V_{FB}.

ii) *(3 points)* The voltages between the nodes D, E, and F are know to have the following values: $V_{DE} = 7\,\mathrm{V}$, $V_{DF} = 15\,\mathrm{V}$, and $V_{EF} = 20\,\mathrm{V}$; what is the value of the input voltage V_0?

8. AIR IN A SUBMARINE (6 points) — *Johan Runeson, Jaan Kalda.* A submarine of unknown nationality is travelling near the bottom of the Baltic sea, at the depth of $h = 300\,\mathrm{m}$. Its interior is one big room of volume $V = 10\,\mathrm{m}^3$ filled with air ($M = 29\,\mathrm{g/mol}$) at pressure $p_0 = 100\,\mathrm{kPa}$ and temperature $t_0 = 20\,°\mathrm{C}$. Suddenly it hits a rock and a large hole of area $A = 20\,\mathrm{cm}^2$ is formed at the bottom of the submarine. As a result, the submarine sinks to the bottom and most of it is filled fast with water, leaving a bubble of air at increased pressure (no air escapes the submarine). The density of water $\rho = 1000\,\mathrm{kg/m}^3$ and free fall acceleration $g = 9.81\,\mathrm{m/s}^2$. Molar heat capacitance of air by constant volume $c_V = \frac{5}{2}R$, where $R = 8.31\,\mathrm{J/Kmol}$ is the gas constant.

i) *(2 points)* What is the volume rate (m^3/s) at which the water flows into the submarine immediately after the formation of the hole?

ii) *(2 points)* The flow rate is so large that the submarine is filled with water so fast that the heat exchange between the gas and the water can be neglected (this applies also to the next question). What is the volume of the air bubble once water flow has stopped?

iii) *(2 points)* The water stream rushing into the submarine creates turbulent water inside the submarine; what is the total kinetic energy of this water turbulence (which later ends up being dissipated as heat), once the inflow has stopped due to equalized pressures?

9. BLACK BOX (?? 1point) — *Jaan Kalda, Mihkel Heidelberg.* The black box has three terminal wires: "blue", "black" and "white", and contains in a star configuration: a battery, a capacitor, an inductor in series with a diode. You may consider the diode to be "ideal" — it conducts current perfectly one way and not at all the other way. You may neglect internal resistance of the battery and capacitor, but the inductor has considerable internal resistance. The multimeter's internal resistance when measuring voltages is $R_m = 10\,\mathrm{M\Omega}$ and it displays a new reading every $t = 0.4\,\mathrm{s}$.

i) *(3 points)* Draw the electrical circuit that is inside the black box. Motivate your solution with measurements.

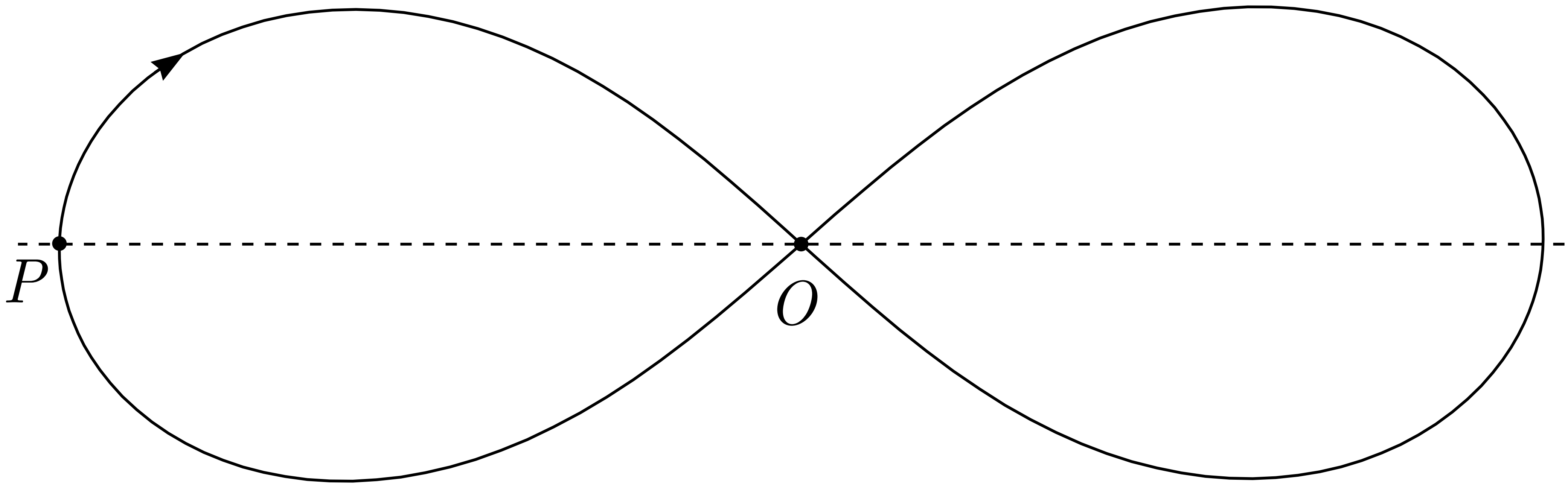
ii) *(1 point)* Determine the electromotive force of the battery.

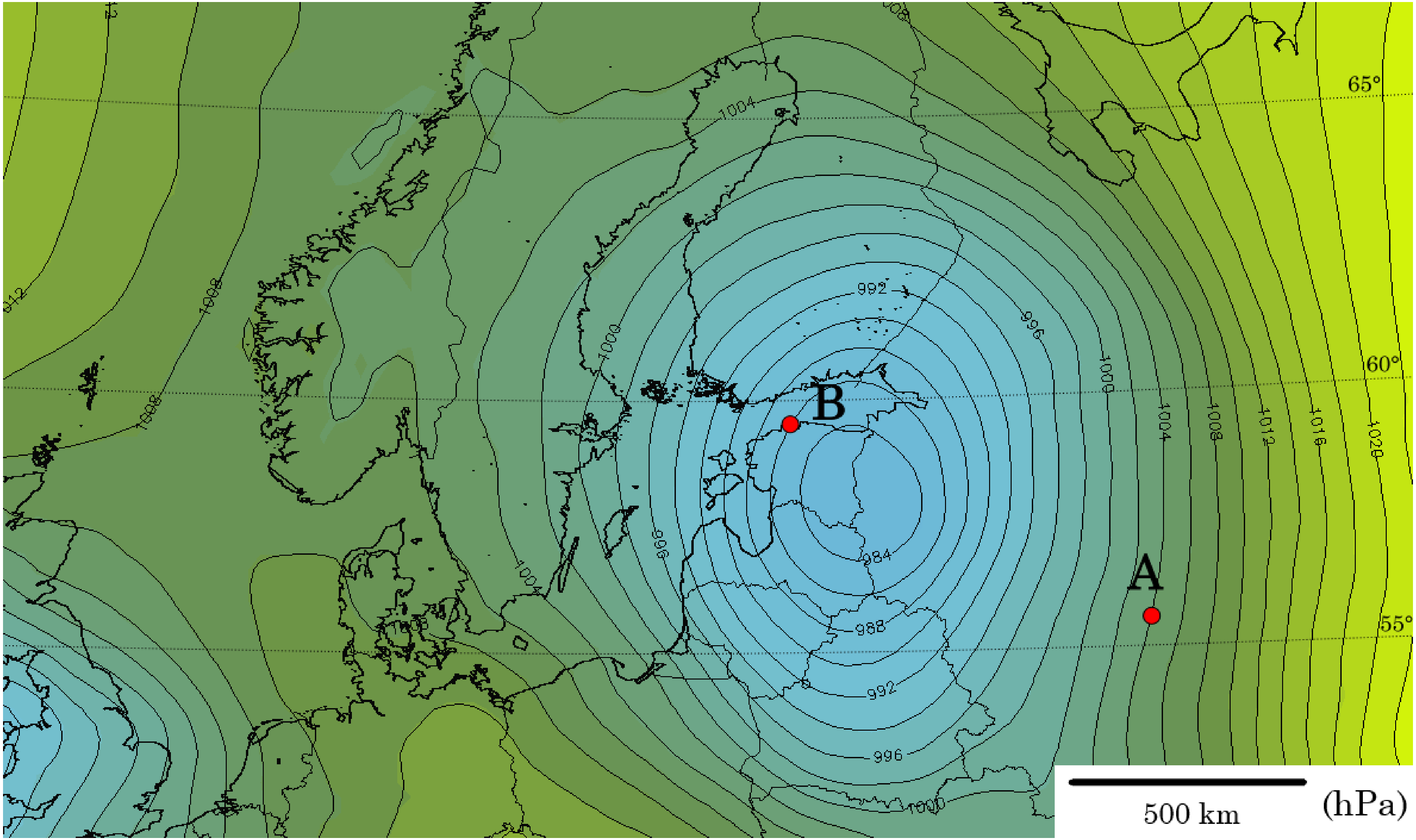
iii) *(1 point)* Determine the internal resistance of the inductor.

iv) *(3 points)* Estimate the value of the capacitance C.

v) *(3 points)* Estimate the value of the inductance L.

Equipment: Black box, multimeter, stopwatch.


P
O



65°
60°
55°
B
A
1004
992
996
1000
1008
1012
1016
1020
984
988
500 km
(hPa)