Nordic-Baltic Physics Olympiad 2017

1. DRAGON (5 points) — *Aigar Vaigu.* Here is a photo of a dragon under water (there is a larger photo on a separate sheet). The length of the dragon is $l = 8\,\text{cm}$ and its height is $h = 3\,\text{cm}$. The diameter of the bottom of the bowl is $d = 10\,\text{cm}$ and the angle between the table and the side of the bowl is $\alpha = 60°$. The refractive index of water is $n = 1.33$. The photo has been taken so that the camera was pointing directly along the water surface. In the following questions the angle between the horizon when looking at some point on the image of the dragon is defined as the angle between the horizontal water surface (or any other horizontal surface) and the straight line from the eye to that point on the image.

i) *(2 points)* What is the largest angle below the horizon from which the reflection of the dragon from the water surface can be seen?

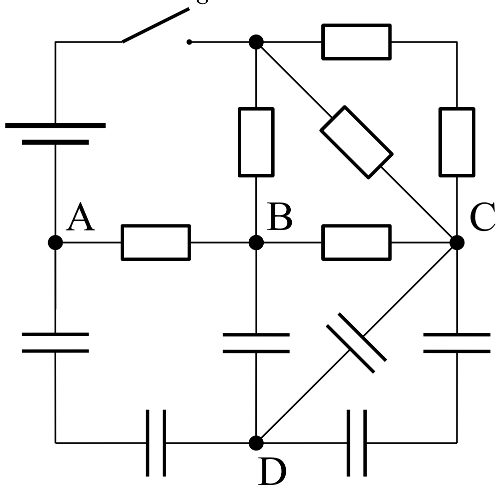
ii) *(3 points)* What is the highest angle above the horizon from which this reflection can be seen?

2. COMET (8 points) — *Jaan Kalda.* A comet's orbit intersects the Earth's orbit (which can be assumed to be circular of radius $R_0 = 1.5 \times 10^8\,\text{km}$) at an angle $\alpha = 45°$. The comet's and the Earth's orbits lie on the same plane.

i) *(3 points)* Find the distance $R_{\min}$ of the comet's perihelion P from the Sun (i.e. its shortest distance). The distance $R_{\max}$ of the comet's aphelion A from the Sun (i.e. its longest distance) can be assumed to be much larger than R_0.

ii) *(5 points)* For how many days t will the comet's distance to the Sun be less that R_0?

3. RESISTORS AND CAPACITORS (5 points) — *Mihkel Heidelberg.*



A circuit is made out of a battery, a switch, resistors and capacitors as shown in the image. The resistors all have a resistance of R, the capacitors all have a capacitance of C and the battery has a voltage of U. The point A is connected to the ground and so it has a potential of 0 V. In the beginning the switch is open and all the capacitors have no charge.

i) *(2 points)* What is the potential at points B and C after we have closed the switch and waited for all the potentials to stabilize?

ii) *(3 points)* What is the potential at point D after we have closed the switch and waited for all the potentials to stabilize?

4. GRAVITATIONAL WAVES (7 points) — *Artūrs Bērziņš.* The power radiated in gravitational waves by an orbiting binary system is given by $P(r, m_1, m_2) = \frac{32}{5}\frac{G^4}{c^5}\frac{(m_1 m_2)^2(m_1+m_2)}{r^5}$, where r is the distance between the centers of the two orbiting masses m_1 and m_2. It is known that the most compact object is a black hole. The size of a black hole is defined by its Schwarzschild radius $r_s = \frac{2Gm}{c^2}$, where m is the mass of the black hole.

i) *(2 points)* Estimate the upper limit of the power that can ever be emitted in gravitational waves by an orbiting binary system.

The gravitational wave detectors on Earth function by measuring the so called gravity wave strain $\varepsilon(t)$ over time, which characterizes the deformation of spacetime. Data processing then yields the maximum strain ε and its corresponding wave frequency f. With the help of a theoretical spacetime model, the energy density u associated with the wave can then be determined. We will use the analogy of linear elasticity to examine this model.

ii) *(1.5 points)* Derive the energy density $u = u(\varepsilon, E)$ in a uniformly stretched elastic band in terms of the strain ε and the Elastic (Young's) modulus E.

iii) *(1.5 points)* Use dimensional analysis to estimate the frequency-dependent elastic modulus of spacetime $E(f)$ in terms of the universal gravitational constant G, speed of light c and gravitational wave frequency f.

iv) *(2 points)* Estimate the maximum distance $z = z(\varepsilon, f)$ from the Earth to the gravitational wave source as a function of the strain ε and frequency f. Use the models derived previously in this problem.

5. VIRTUAL MASS (10 points) — *Jaan Kalda.* When a body moves in a fluid, the effective inertial mass appears to be larger than the mass of the body itself, because to accelerate the body some of the fluid needs to be accelerated as well. This increase is called added mass or virtual mass. Measure the amount of virtual mass m_v that is added to the ball as it moves through water. The diameter of the ball is $d = 72.0\,\text{mm}$. You don't have to calculate the uncertainty of your results, however the accuracy of your methods and results is important and will be graded.

Equipment: Ball fixed to a spring, stand, stopwatch, ruler, container with water.

6. LOOP (6 points) — *Lasse Franti.* A wire loop is hovering in outer space (weightless vacuum) with its plane parallel to the xy-plane. In $x < 0$ there is a homogeneous magnetic field parallel to the z-axis. The rigid rectangular loop is $l = 10\,\text{cm}$ wide and $h = 30\,\text{cm}$ long. The loop is made of copper wire with a circular cross section (radius $r = 1.0\,\text{mm}$). At $t = 0\,\text{s}$ the external magnetic field starts to decrease at a rate of 0.025 T/s.

i) *(3 points)* Find the acceleration of the loop right after $t = 0\,\text{s}$. The magnetic flux density is initially $B = 2.0\,\text{T}$ and the loop is immersed $d = 12\,\text{cm}$ into the external field with its shorter side parallel to the y-axis.

ii) *(3 points)* We can try to increase the acceleration in many ways. How does the result in i) change if

a) the loop is made with twice thicker copper wire ($r = 2.0\,\text{mm}$)?

b) the original wire is wound three times instead of one (giving a short-circuited coil with three windings, the dimensions of the rectangle stay the same)?

c) the mass of the coil is kept constant by using a wire with half the cross section which is then wound two times (the dimensions of the rectangle stay the same)?

d) the loop is made of a different metal? Which of the metals given in the table gives the best result?

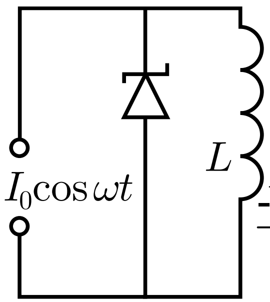
Metal	Resistivity $10^{-8}\,\text{m}$	Density $10^3\,\text{kg/m}^3$
Iron	9.71	7.87
Copper	1.67	8.96
Aluminum	2.65	2.70
Lithium	8.55	0.53

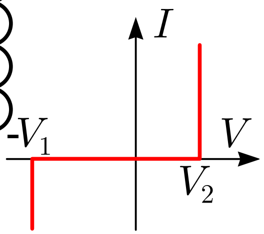
e) What if we use the thicker copper wire to make a twice bigger loop ($r = 2.0\,\text{mm}$, $l = 20\,\text{cm}$, $h = 60\,\text{cm}$) and immerse it 24 cm into the external field?

7. ZENER (7 points) — *Jaan Kalda*. A Zener diode is connected to a source of alternating current as shown in the figure. The current is sinusoidal $I = I_0 \cos\omega t$ with a constant amplitude. The inductance L of the inductor is such that $L\omega I_0 \gg V_1, V_2$, where V_1 and V_2 are the breakdown voltages ($V_1 > V_2$). The current-voltage characteristic of the Zener diode is shown in the figure. In the following assume that a very long time has passed since the current source was first turned on.

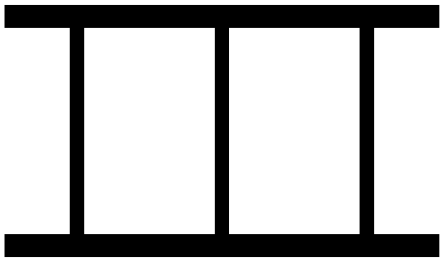
i) *(5 points)* Find the average current $\langle I\rangle$ through the inductor.

ii) *(2 points)* Find the peak-to-peak amplitude of the current changes ΔI in the inductor.





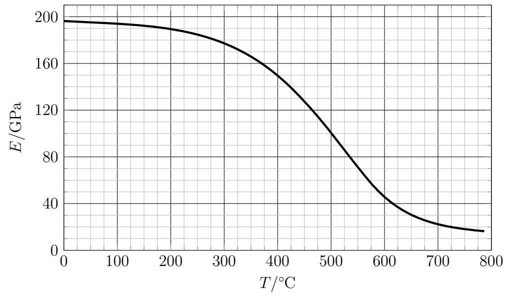
8. BEAMS (6 points) — *Andres Põldaru*. There are three beams between two absolutely rigid plates. The weight of the plates and the beams can be neglected. The coefficient of thermal expansion of the beams is $\alpha = 1.0\times 10^{-5}\,\mathrm{K}^{-1}$. The maximum strain (relative length change compared to the no load case) before permanent inelastic deformations of the material of the beam is $\beta = 0.40\%$. The beams can support a maximum amount of weight on the top plate, at which point permanent deformations start taking place in some of the beams.



i) *(2 points)* Initially all the beams are at the same temperature. Then the temperature of the beam in the middle is increased by $\Delta T = 100\,\mathrm{K}$. Compared to the case when the beams were at equal temperatures, what fraction of the original maximal weight can the beams now support on the top plate? Assume that the properties of the materials (specifically the maximum strain and the elastic modulus) don't change during the heating.

ii) *(4 points)* All of the beams are originally at temperature $T_0 = 0\,°\mathrm{C}$. A load of 20% of the maximal load is placed on the top plate. Keeping the same load on, to what temperature can the beam in the middle be heated such that no permanent deformations occur? This time the elastic modulus of the material changes depending on the temperature as shown in the figure below (a larger figure is given on an extra sheet).

Hint (added during the competition): the elastic modulus or Young modulus E is defined by the formula $F/A = E\Delta l/l$, where F is the force, A is the area and $\Delta l/l$ is the relative lengthening.



9. SPACECRAFT PRESSURE (6 points) — *Johan Runeson*. Consider a spacecraft shaped as a homogeneous pipe, which is closed at both ends. The spacecraft is rotating around its center of mass with angular velocity ω, around an axis perpendicular to the pipe, in order to simulate gravity. The spacecraft is filled with air of molar mass μ, which has pressure p_0 at the rotation axis. The diameter of the spacecraft is much smaller than its length. *Added during the competition: the temperature is T.*

i) *(4 points)* Calculate the pressure p as a function of the distance r from the rotation axis.

ii) *(2 points)* As a comparison, consider a (non-rotating) tower in a constant gravitational field of strength g, filled with the same gas. If the ground level pressure is p_0, what is the pressure p as a function of height h above the ground in such a tower?

10. BLACK BOX (10 points) — *Jaan Kalda, Siim Ainsaar*. In a black box with three terminals (A, B and C) there is a resistor (resistance R_1), a capacitor (capacitance C), and a series connection of a battery (electromotive force $\mathscr{E}$) with another resistor (R_2).

i) *(3 points)* Determine the circuit diagram of the black box.

ii) *(7 points)* Measure the electromotive force of the battery, the resistances of the resistors, and the capacitance of the capacitor. Estimate the uncertainties. Always indicate the circuit diagram and the multimeter settings you used for the measurements!

Equipment: black box, multimeter, stopwatch, wires, graph paper.