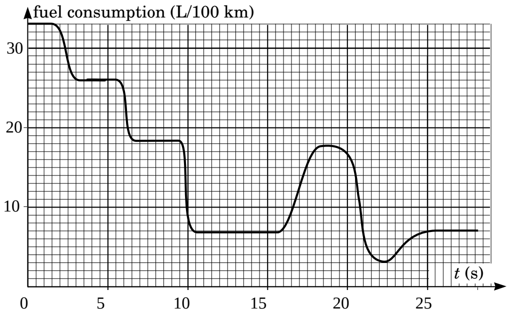
Estonian-Finnish Olympiad 2016

1. FUEL CONSUMPTION (5 points) — *Jaan Kalda.* The given graph (a larger copy is on an extra sheet) shows a car's fuel consumption as a function of time. It is known that the car started moving from a horizontal road segment with initial acceleration $a_0 = 5\,\mathrm{m/s^2}$. At time $t_1 = 11\,\mathrm{s}$, the road was horizontal and the driver switched on the cruise control for constant speed $v_0 = 90\,\mathrm{km/h}$. Shortly afterwards, the car started moving up a hill. How high was the highest point on the road over that hill, relative to the height at t_1? Assume that the efficiency of the car was constant at all times. Remark: between seconds 2 and 10, there might have been ascents and/or descents on the road.



2. GLASS PLATE (10 points) — *Rasmus Kisel and Mihkel Heidelberg.* Light from a laser with a wavelength of λ and power of P falls perpendicularly on to a thin glass plate of thickness $a = (100.25\lambda)/n$ where n is the index of refraction of glass. The reflection coefficient of the glass surfaces is r, which can be interpreted as the probability for a photon to be reflected from a single glass surface.

i) *(2 points)* Calculate the force F_a applied to the glass by the laser light when the incident side of the glass is painted totally black.

ii) *(3 points)* Calculate the force F_b applied to the glass by the laser light when the opposite side of the glass is painted totally black.

iii) *(5 points)* Calculate the force F_c applied to the glass by the laser light when neither of the sides is painted.

3. MUSIC (8 points) — *Lasse Frantti.* A band consisting of three physicists is on tour. The band plays progressive world music and is equipped with an electric guitar, pipe organ and tubular bells made of steel. Their first gig is at Tavastia-club in Helsinki, where they tune their instruments before the show. The air is comfortably dry and the temperature is 25 degrees Celsius.

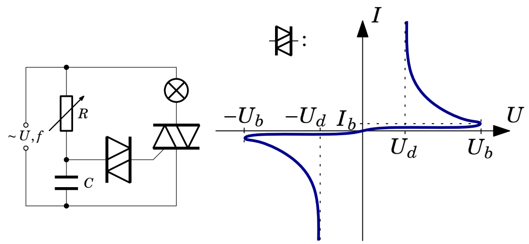
i) *(6.5 points)* Their second gig is in Libya, where the temperature is 45 degrees Celsius. Their instruments are out of tune because of the scorching heat but all the tuning equipment has been left home. How badly out of tune are they? Estimate the audible change in the original 330 Hz tuning in all of the three instruments.

ii) *(1.5 points)* The tour ends with a private gig at a pulmonary clinic. The temperature in the treatment room is 25 degrees, but instead of air the room is filled with a mixture of helium and air (heliox). How does this affect the audible sound produced by the three instruments?

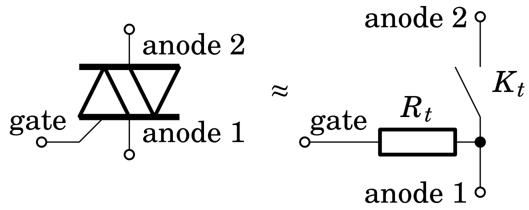
Speed of sound in air
$v_a = 331.3\,\mathrm{m/s}\sqrt{1 + \frac{t(^\circ\mathrm{C})}{273.15}}$
Speed of sound in heliox $v_t = 1.7 v_a$
Heat capacity of steel $450\,\mathrm{J/kg\cdot K}$
Density of steel $7900\,\mathrm{kg/m^3}$
Melting point of steel $1540\,^\circ\mathrm{C}$
Heat conductivity of steel $50\,\mathrm{W/mK}$
Young modulus of steel 200 GPa
Coefficient of thermal expansion (steel) $12.0\times10^{-6}\,\mathrm{K^{-1}}$
Power of the guitar amplifier 500 W
Diameter of the guitar E string 0.30 mm
Free length of the E string 65 cm
Frequency of the E string 330 Hz

You can neglect the effects of temperature on the guitar body. The guitar string pitch is determined by the transverse standing wave frequency on the guitar string. The pipe organ pitch is determined by the longitudinal standing wave frequency of air in the pipe. The tubular bell pitch is determined by the transverse standing wave of the (steel) tube. You may use dimensional analysis where possible. Assume that the Young modulus of steel is constant in temperature.

4. DIMMER (9 points) — *Siim Ainsaar.* A dimmer for controlling the brightness of lighting consists of a rheostat, a capacitor, a diac and a triac, connected as in the schematics.



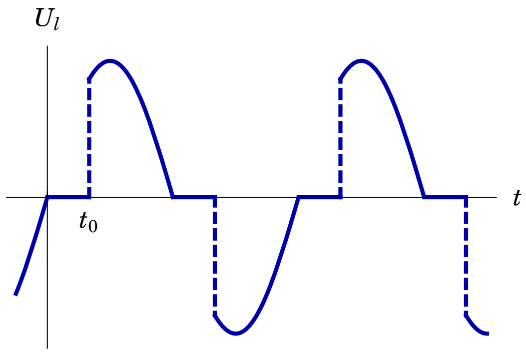
A diac is a component whose behaviour is determined by the voltage-current diagram shown above. A triac, on the other hand, can be thought of as a switch controlled by current — look at the following equivalent schematics.



The switch K_t is open as long as the current through the triac's gate stays under the threshold current I_t; closes when the threshold current is applied (in either direction) and stays closed while a current is flowing through the *switch* K_t (the gate current is irrelevant until the switch opens again).

i) *(3 points)* Assume that the resistance R_t is large enough that the charge moving through the diac can be neglected. Let the sinusoidal supply voltage have a maximum value of U and a frequency of f; the rheostat be set to the resistance R and the capacitor's capacitance be C. Find the maximum value of the voltage U_C on the capacitor, and its phase shift φ with respect to the supply voltage.

ii) *(2 points)* What inequality should be satisfied by the diac's characteristic voltages U_b and U_d, triac's threshold current I_t and gate resistance R_t to ensure that when the diac starts to conduct (while the voltage on the capacitor rises), then the triac would also immediately start to conduct? You may assume that $I_b < I_t$ and that the diac's voltage at current I_t is U_d.

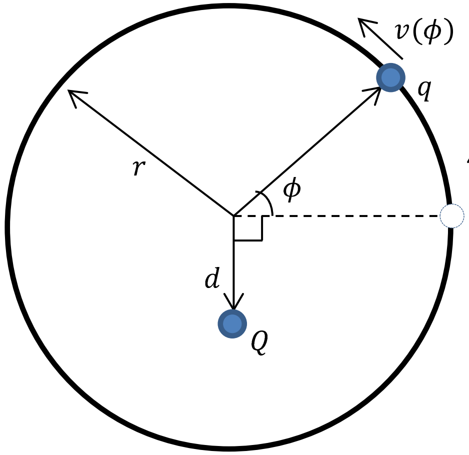


iii) *(2 points)* The voltage U_l on the lamp follows the plot above. Let's assume that the assumption of part i) and the inequality of part ii) hold. Find the time t_0 during which the voltage on the lamp is zero.

iv) *(2 points)* Express through t_0 and f, how many times the average power of the lamp is lower than the one of a lamp without a dimmer, assuming that the resistance of the lamp is unchanged.

5. CANDY WRAPPER (6 points) — *Eero Uustalu.* Measure the thickness d of the candy wrapper, estimate the uncertainty. *Equipment:* Candy, two hexagonal pencils, rubber bands, green $\lambda = 532\,\mathrm{nm}$ laser, measuring tape, screen, stand. **Warning: do not look into the laser beam and do not direct it to the eyes of others!**

1. **CHARGE ON A RING (7 points)**— *Andreas Isacsson.* A pointlike particle with mass m and charge q is free to slide without friction along a fixed horizontal circular ring with radius r. In the plane of the ring, another charge Q is placed in a fixed position, at a distance d from the center of the ring, with $d < r$ (see figure).

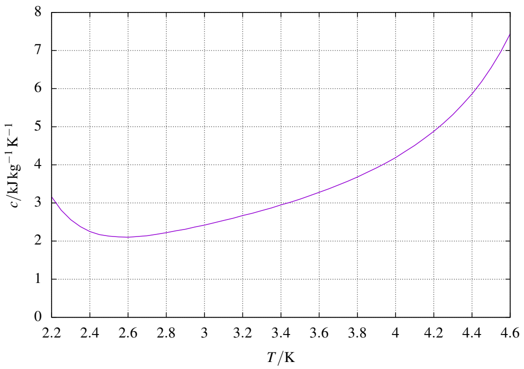


i) *(3 points)* The particle on the ring is given an initial velocity v_0. Calculate its velocity v as a function of the angle ϕ, $v(\phi)$.

ii) *(2 points)* How large is the force from the ring on the particle, as a function of the angle ϕ?

iii) *(2 points)* Viscous friction can be modelled with a force directed against the velocity, with a magnitude proportional to the speed, i.e. $|F_f| = m\gamma v$, where γ is a positive constant. Assume that this kind of frictional force acts on the particle on the ring. Given an initial v_0, determine the position where the particle comes to rest.

2. **HELIUM (6 points)**—*Jaan Toots.* Liquid helium is cooled under low pressure by vaporizing it and pumping the gas away. The heat of vaporization of helium is $\lambda = 22\,\mathrm{kJ\,kg^{-1}}$, which you can take to be constant. The specific heat of the liquid $c(T)$ is shown on the graph (a larger copy is on an extra sheet). What fraction of the liquid helium has to vaporize to cool the remaining liquid from $T_0 = 4.1\,\mathrm{K}$ to $T_1 = 2.3\,\mathrm{K}$?



3. **OSCILLATIONS (7 points)**— *Lasse Frantti.*

i) *(2 points)* A steel ball (mass $m = 1\,\mathrm{kg}$) is attached to an ideal vertical spring. This causes the spring to lengthen by $x = 5\,\mathrm{cm}$. The ball is then pulled down by $s = 10\,\mathrm{cm}$ from its equilibrium point and released, which leads to an oscillation. What should be the length of a point mass pendulum in order to have the same small oscillations period as this system?

ii) *(2 points)* A hole is drilled diametrically through a spherical asteroid. An astronaut drops a stone into the hole to see what happens. Show that the stone starts to oscillate harmonically back and forth in the hole.

iii) *(2 points)* You want to send a parcel to your friend at the other end of the drilled hole. You can either throw the parcel horizontally (around the asteroid) or drop it into the hole. Which way is faster?

iv) *(1 point)* A perfectly elastic ball is dropped onto a horizontal table from height $h = 50\,\mathrm{cm}$. Estimate the period of bounces. Is this motion a simple harmonic oscillation? Why/why not?

4. **DEFLECTION ON FALLING (8 points)** — *Mihkel Kree.* Imagine a vertical mine shaft of height $h = 100\,\mathrm{m}$ at the equator. Consider a free falling steel ball, released from the top of the shaft, and let us neglect the air drag (friction). Not surprisingly, due to the rotation of the Earth, the ball will reach the bottom of the shaft at a point which is slightly different from the point of the vertical projection of the release location. Let us denote the distance between these points by Δx. (Historical note: the correct expression for Δx was first calculated independently by Laplace and Gauss in 1803.)

i) *(1 point)* Calculate the velocity difference Δv of the top of the shaft and the bottom of the shaft as seen in an absolute, non-rotating frame of reference.

ii) *(1 point)* Neglecting the rotation of the Earth, but assuming that the steel ball is released with an initial horizontal velocity Δv as seen from the bottom of the shaft, calculate the horizontal displacement Δx_0 of the steel ball by the time it reaches the bottom.

Obviously, the answer found in ii) is not physically correct, because we neglected the rotation of the Earth and, correspondingly, the Coriolis force. Luckily, the correct expression can still be found without integrating the Coriolis force (although Laplace and Gauss did it).

iii) *(6 points)* Considering the falling steel ball being on a Keplerian orbit and taking into account the curvature of the Earth, find the true value of the horizontal displacement Δx. *Hint: use appropriate approximations for small values and note that the small segment of ellipse corresponding to the fall of the steel ball can still be approximated as a parabola.*

5. **BLACK BOX (10 points)**— *Mihkel Heidelberg and Jaan Kalda.* The black box has three terminal wires: "blue", "black" and "white", and contains two resistors with resistances $R_1 < R_2 < 1\,\mathrm{k\Omega}$ and a single Zener diode in some arrangement. In the voltage ranges we are using, the Zener diode behaves as a regular diode — conducts current well when voltage is applied in the forward direction, and conducts little when voltage is applied in the reverse direction. We are using a Zener diode, because it has a higher leakage current of up to 1 mA when voltage is applied in reverse direction. You may neglect the internal resistance of the voltmeter in all ranges and of the ammeter in the 40 mA and 400 mA ranges. The internal resistance of the ammeter in the 400 µA and 4000 µA ranges is $R_A = 100\,\Omega$.

i) *(2 points)* Draw the electrical circuit that is inside the black box. Motivate your solution with measurements.

ii) *(4 points)* Measure the resistances R_1 and R_2, estimate the uncertainties.

iii) *(4 points)* Measure and draw the current-voltage curve of the Zener diode in the black box, use as many different datapoints as possible.

Equipment: Black box, DC voltage source, multimeter.

Do not anger Eero by shorting the voltage source with the ammeter, you will blow the fuse.