Estonian-Finnish Olympiad 2015

Editors: S. Ainsaar, M. Heidelberg

1. ANNIHILATION (6 points) *(by L. Franti)* An electron with kinetic energy 1 MeV travels along the z-axis and collides with a positron at rest. The particles annihilate producing a pair of two photons A and B with equal energies.

i) *(1.5 points)* Find the speed of the electron v_e in the lab frame.

ii) *(0.5 points)* Find the energy E_γ of photon A in the lab frame.

iii) *(1 point)* Find the momentum p_γ of photon A in the lab frame.

iv) *(2 points)* Find the angle α between the z-axis and the momentum of photon A in the lab frame.

v) *(1 point)* Why is it not possible for a collision process to produce just one single photon?

The rest mass of electron $m_e = 511\,\mathrm{keV}/c^2$. You can express your results in electron volts and related units.

2. HOLOGRAPHIC LENS (7 points) *(by J. Kalda)* Monochromatic light can be focused into a point using an holographic lens (better known as a *Fresnel zone plate*). This is a thin film which has a system of opaque and transparent concentric rings: the opaque rings obstruct those light waves which would arrive at the focus in the opposite phase (as compared with the light from the transparent rings). In what follows, let us assume that the diameter and focal length of such a lens $d = f = 10\,\mathrm{cm}$; the wavelength of the monochromatic incident light radiation is $\lambda = 5\times10^{-7}\,\mathrm{m}$.

i) *(1.5 points)* Starting enumeration from the (opaque) centre of the lens, what is the radius of the m-th transparent ring (more precisely, the circle at the middle of the ring)?

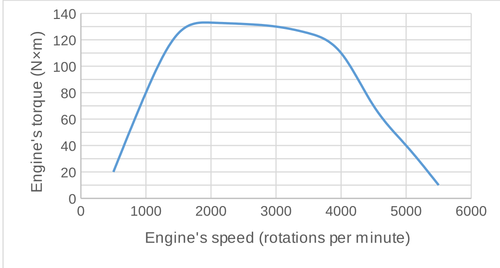
ii) *(1.5 points)* Let us consider also a glass lens of the same focal length f and diameter d, such that a parallel incident beam is collected perfectly into the focus (has aspheric surfaces). What is the minimal possible thickness of such a lens if the coefficient of refraction of the glass is $n = 1.5$?

iii) *(2 points)* If a short light pulse falls onto a lens, the behaviour of the holographic lens is very different from that of the glass lens. Indeed, if we ignore the dispersion of the glass (the dependence of n on the wavelength), the pulse duration at the focus of the glass lens is the same as that of the incident beam. Meanwhile, in the case of a short pulse ($\tau = 3\times10^{-14}\,\mathrm{s}$), the pulse duration at the focus of the holographic lens becomes significantly dilated. Sketch qualitatively the light intensity I at the focus of the holographic lens as a function of time. A scale is required for time t, but not for I. The speed of light $c = 3\times10^8\,\mathrm{m/s}$.

iv) *(1 point)* Even if the light source is an ideal laser providing a perfectly monochromatic radiation, the short pulse is no longer strictly monochromatic. Estimate the width of the range of wavelengths in the pulse of duration τ.

v) *(1 point)* Estimate the duration of this light pulse in the focus of the glass lens. Assume that in the glass, the light wave group velocity v_g depends on the wavelength at the rate $\frac{\mathrm{d}v_g}{\mathrm{d}\lambda} = 0.02\times\frac{v_g}{\lambda}$ and that for the wavelength λ, group speed is equal to the phase speed.

3. GEAR SHIFT (3 points) *(by K.A. Saar)* The maximum torque of a car's engine depends on the engine's rotation speed (see the following figure, a larger copy is on an extra sheet).



The rotations of the engine are transferred to the wheels through a transmission gearbox. When the car is in the first gear, then the gear ratio from the engine to the wheels is 14:1; in the second gear, the gear ratio is 7:1.

i) *(2 points)* At what speed of the car should the transmission be shifted from the first gear into the second, in order for the average acceleration of the car to be maximum?

ii) *(1 point)* What is the acceleration of the car immediately before and immediately after the gear change?

Neglect any drag force. The mass of the car $m = 1400\,\mathrm{kg}$, the diameter of a wheel is $d = 60\,\mathrm{cm}$.

4. STAR WARS (8 points) *(by J. Kalda)* Here we'll study, in which region of space targets can be hit with an interplanetary ballistic missile. The launching point P is motionless in the inertial frame of reference of a gravitationally attracting star centred at S. The launching speed of the missile is fixed throughout this problem and equal to v_0, the same applies to the distance to launching point $|SP| = R$. The mass of the star is M, and the gravity constant is G. *Hint:* the total energy of a missile of mass m moving on an elliptical orbit is $E = -GMm/(2a)$, where a is the longer semiaxis of its orbit.

i) *(2 points)* The missile is launched as described above in an arbitrary direction. What is the period of the missile on its elliptic orbit?

ii) *(2 points)* According to the Kepler's I law, the missile's orbit is an ellipse, with one of the foci being at S. The position of the other focus F depends on the launching angle. While the launching angle is changed, the focus F moves along a certain curve; what curve is it? Also, determine the geometric dimensions of that curve.

iii) *(1 point)* Let us consider such a point Q on the missile's trajectory, the distance of which from the star equals to $|SQ| = r$. Determine the distance $|QF|$.

iv) *(1 point)* The position of point Q (as defined above) depends on the launching angle of the missile, and so does the distance $|PQ|$. Among all the possible positions of the point Q, what is the largest possible distance $|PQ|$?

v) *(2 points)* For questions iii and iv, we assumed that the distance r had a fixed value. Now, if we consider r as a free parameter, the position of the point Q is a function of both r and of the launching angle. What is the boundary of that region of space to which the point Q can belong?

5. RADIATOR (8 points) *(by M. Heidelberg)* Measure the heat transfer coefficient h between the environment and an aluminium profile. $h = \frac{p}{T-T_0}$, where p is the heat flux from the aluminium profile to air per unit of length; T and T_0 are the temperatures of aluminium and the environment, respectively. For measurements, keep the aluminium profile flat on your table surface, to limit nonlinear effects from turbulence. Thermal conductivity of aluminium is $k = 205\,\mathrm{W/(K\cdot m)}$ and the cross-section of the profile is $A = 36\,\mathrm{mm}^2$.

Hint: to describe the stationary temperature distribution along the profile we can derive a one-dimensional Helmholtz equation $T''(x) = \frac{h(T(x)-T_0)}{kA}$, the general solution of which is

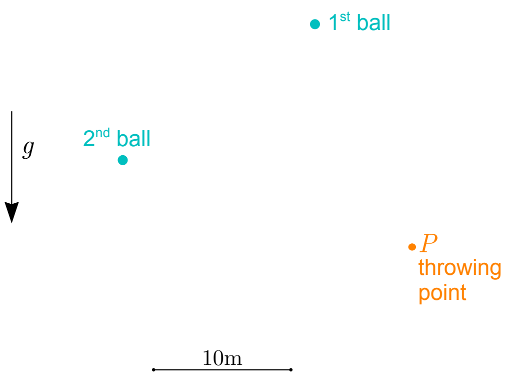
$$T(x) = T_0 + C_1 e^{x\sqrt{\frac{h}{kA}}} + C_2 e^{-x\sqrt{\frac{h}{kA}}},$$

where C_1 and C_2 are integration constants.

Equipment: Aluminium profile, wire (for heating), DC power source, infrared thermometer, ruler.

Keep the temperature of the wire below 150 °C as both the wire insulation and the table surface will start to emit smoke after that point!

6. TWO BALLS (5 points) *(by J. Kalda)* The following snapshot (a larger version is on an extra sheet) depicts two balls that were thrown simultaneously and with the same initial speed, but in different directions from point P. What was the initial speed? Use $g = 9.8\,\mathrm{m/s^2}$.



7. BOUNCY BALL (5 points) *(by J. Toots)* A homogeneous elastic ball with radius R bounces from a vertical wall in such a way that it rebounds back exactly in the same direction it came from. The ball's speed before the bounce is v at an angle α to the vertical; the ball also has an angular speed ω. There was no slipping at the contact point, but do not assume that $\omega R = v\cos\alpha$. The bounce was completely elastic, ie no kinetic energy was lost; also, the horizontal components of incident and rebound velocities have the same magnitude. The moment of inertia for a homogeneous ball is $I = \frac{2}{5}mR^2$.

i) *(1 point)* Find the angular speed ω_2 of the ball after the bounce.

ii) *(2 points)* Find the initial angular speed ω in terms of v, α and R.

iii) *(2 points)* What is the minimum value for coefficient of friction μ that such bounce could happen?

8. ELECTRIC FIELD (6 points) *(by L. Franti)* A conducting ring with radius R carries a large current I. The ring lies stationary in the xy-plane with its centerpoint at (0,0,0). An observer approaches the ring parallel to the z-axis with velocity v and measures the electric field at distance r from the axis. Assume that $v \ll c$ and $r \ll R$.

i) *(2 points)* Find the direction and magnitude of the magnetic field $B(z)$ on z axis in stationary reference frame.

ii) *(4 points)* Estimate the magnitude and direction of electric field $E(z,r)$ measured by the observer when he is at a distance z from the ring's plane.

9. SOLENOIDS (8 points) *(by S. Ainsaar).* A tightly-wound rigid solenoid coil has been partly inserted into another. They are connected to a constant current source so that the same current I flows through them; both generate a magnetic field in the same direction. The solenoids have N turns both, their length is l, their cross-sectional areas are A_1 and A_2. It may be assumed that $A_1, A_2 \ll l^2$. You may find it useful to know that the magnetic field at the centre of one solenoid taken separately is $B = \mu_0 IN/l$, where μ_0 is the permeability of vacuum.

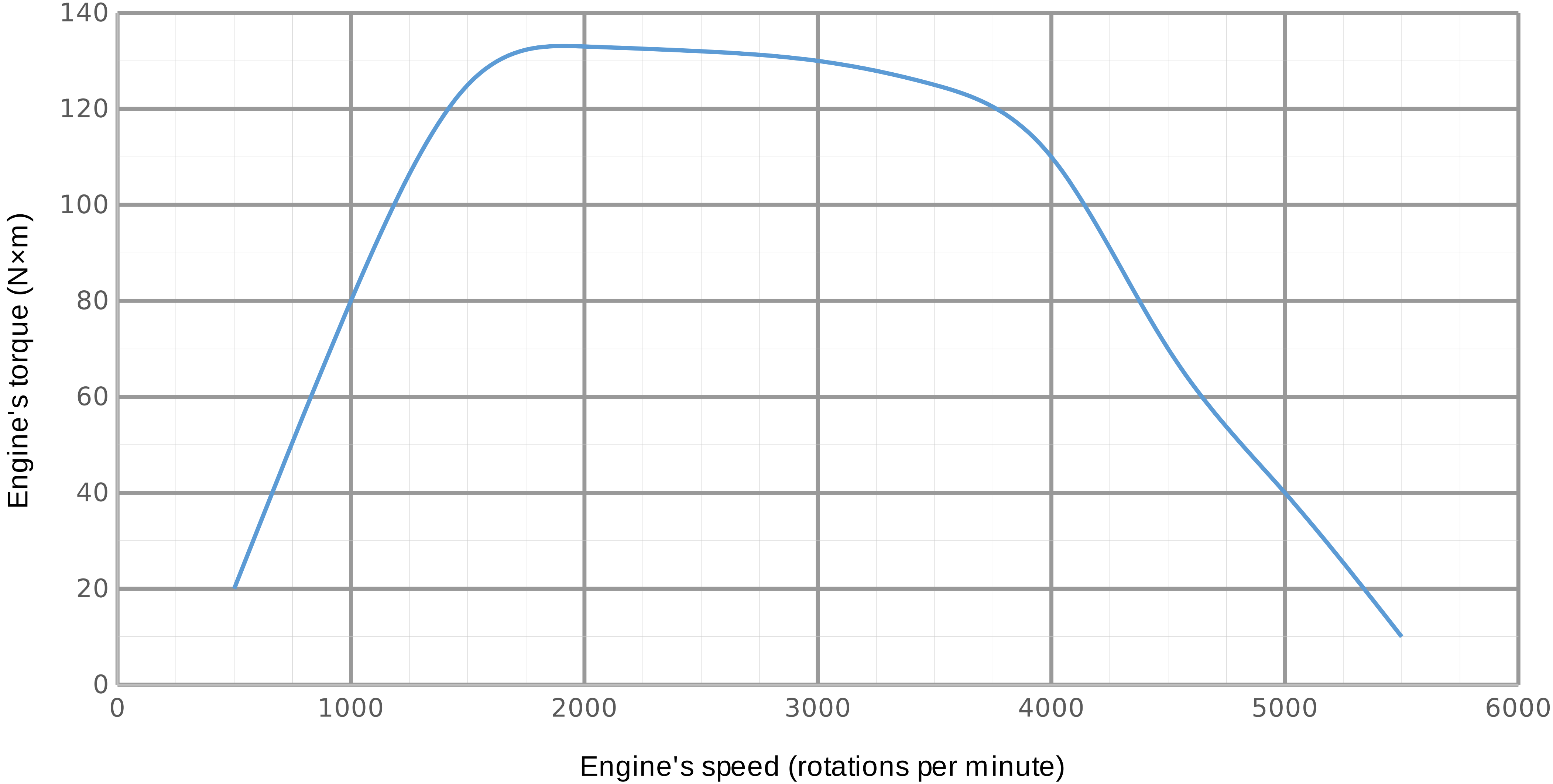
i) *(2 points)* The centres of the coils are at a distance $x < l$ along their common axis $[A_1, A_2 \ll (x-l)^2, x^2]$. Find the total energy E_m of the magnetic field in the system.

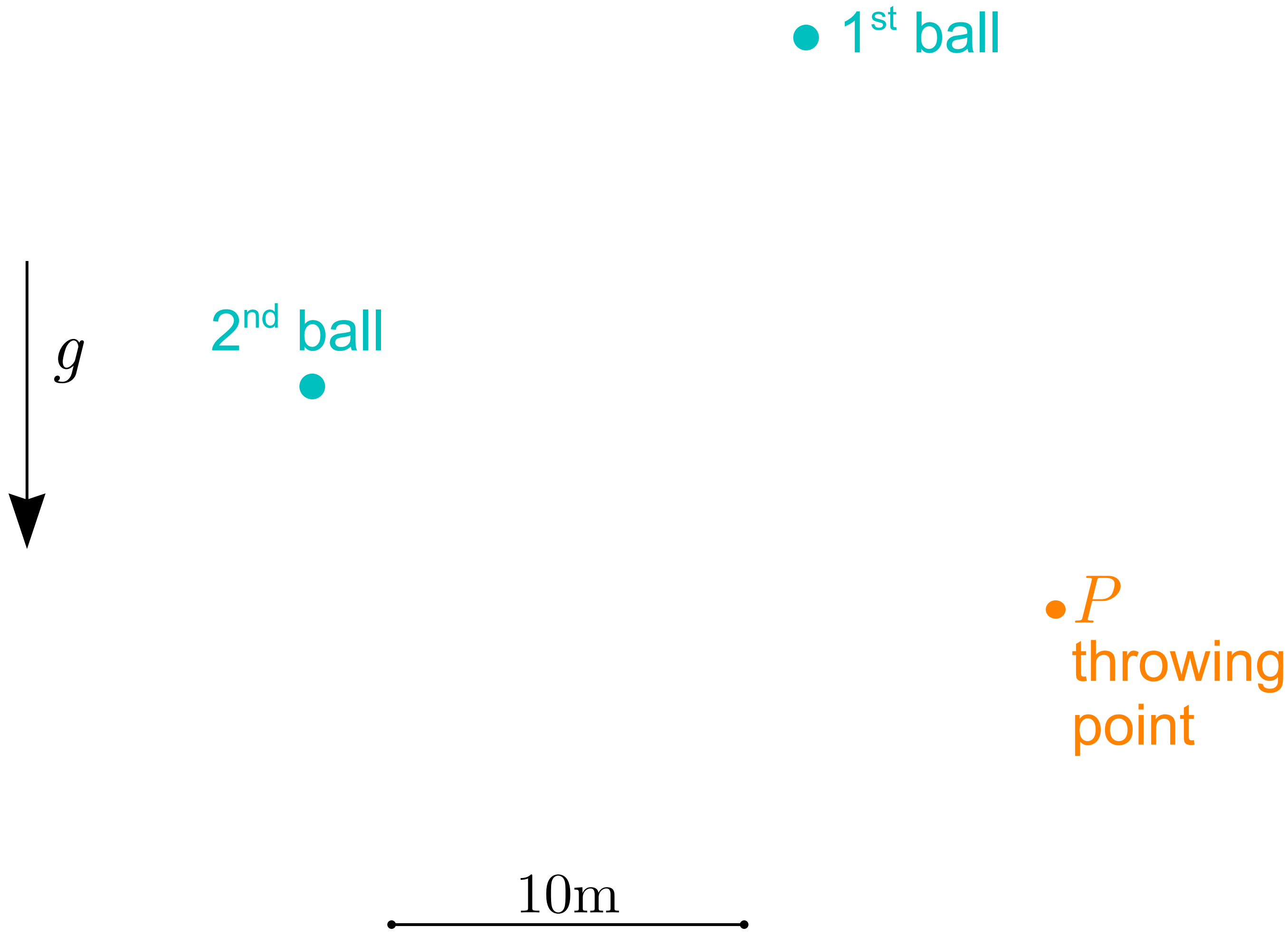
ii) *(4 points)* Find the electromotive forces $\mathscr{E}_1$ and $\mathscr{E}_2$ generated on the coils when one is pulled out with a speed v.

iii) *(2 points)* Find the force F needed to pull one coil outwards.

10. VAPOUR PRESSURE (8 points) *(by J. Kalda, M. Heidelberg, E. Uustalu)* Measure the room temperature saturated vapour pressure of the unknown fluid in the syringe. The volume of the bottle is $V = 0.50\,\ell$, the inner diameter of the pipe is $d = 6.0\,\mathrm{mm}$. Current air pressure p_0 and room temperature T_0 are written on the blackboard. There is no need to repeat the measurement after a successful attempt as getting the bottle free from vapours is time consuming. Estimate the uncertainty of the result.

Equipment: syringe with an unknown fluid, syringe with water, bottle, flexible pipe, ruler, plug, stand, tape.


140
120
100
80
60
40
20
0
0
1000
2000
3000
4000
5000
6000
Engine's torque (N×m)
Engine's speed (rotations per minute)



1st ball
2nd ball
g
P
throwing point
10m