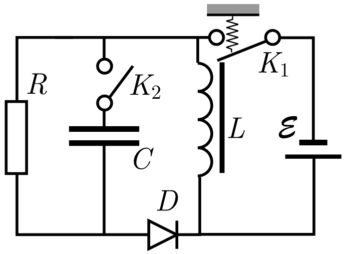
Estonian-Finnish Olympiad 2014

1. DC-DC CONVERTER (8 points) In order to obtain high voltage supply using a battery, the following circuit is used.



An electromagnetic switch K_1 connects a battery of electromotive force $\mathscr{E}$ to an inductor of inductance L: it is closed if there is no current in the inductor (a spring keeps it closed), but if the inductor current reaches a critical value I_0, magnetic field created by the inductor pulls it open. Due to inertia, once the key is open, it takes a certain time τ_K to close again even if the current falls to zero.

For the diode D you may assume that its current is zero for any reverse voltage ($V_D < 0$), and also for any forward voltage smaller than the opening voltage V_0 (i.e. for $0 < V_D < V_0$). For any non-zero forward current, the diode voltage V_D remains equal to V_0.

You may express your answers in terms of L, $\mathscr{E}$, I_0, V_0, and the capacitance C (see figure).

i) *(1 point)* At first, let the key K_2 be open. If the initial inductor current is zero, how long time τ_L will it take to open the key K_1?

ii) *(1 point)* Assuming (here and in what follows) that $L/R \ll \tau_K \ll \tau_L$, plot the inductor current as a function of time t (for $0 \le t < 3\tau_L$).

iii) *(1 point)* What is the maximal voltage $V_{\max}$ on the resistor R?

iv) *(2 points)* Assuming that $V_{\max} \gg V_0$, what is the average power dissipation on the diode?

v) *(2 points)* Now, let the key K_2 be closed, and let us assume simplifyingly that $V_0 = 0$; also, $RC \gg \tau_L$ and $\tau_K > \pi\sqrt{LC}$. Suppose that the circuit has been operated for a very long time. Find the average voltage on the resistor.

vi) *(1 point)* Find the amplitude of voltage variations on the resistor.

2. WASTE PROJECT (8 points) In 2114, Europarliament decided that all radioactive wastes need to be sent to the Sun, so as to avoid contamination of Earth and orbital space. In what follows, you can use the following numerical data: duration of one year $T = 365.25$ days, orbital speed on Earth $v_0 = 29.8$ km/s, angular diameter of Sun as seen from the Earth $\alpha = 0.5°$, radius of the Earth $R = 6400$ km, free fall acceleration at the Earth's surface $g = 9.81\,\mathrm{m/s^2}$.

According to the project, the waste is sent to the Sun using ballistic spaceships: the engine operates only during a short period of time during which the displacement of the spaceship remains much shorter than the radius of Earth. In the Earth's frame of reference, the spaceship obtains a velocity opposite to the orbital velocity of Earth in the Sun's frame of reference. Further, the ship moves along a ballistic trajectory until it hits the Sun. The trajectory is such as to minimize the consumption of fuel.

i) *(1 point)* Sketch the trajectory of the spaceship.

As a first approximation, calculations can be done when neglecting the angular size of the Sun (i.e. by putting $\alpha \approx 0°$); you can use this approach for the next two questions.

ii) *(1.5 points)* How long will it take for the spaceship to travel from the Earth to the Sun?

iii) *(1.5 points)* What is the speed of the spaceship in the Earth's frame of reference when the distance from the Earth is much larger than the Earth's radius, but still much shorter than the distance to the Sun?

iv) *(2.5 points)* Answer the previous question without making the approximation $\alpha \approx 0°$.

v) *(1.5 points)* What is the speed of the spaceship in the Earth's frame of reference when the distance from the Earth is much smaller than the Earth's radius?

3. MAGNETS (6 points) To explore the force between two small magnets, the following experiment is performed. One of the magnets is hanged from a thread with length $l = 1$ m. Other magnet is moved slowly closer while keeping the axes of the magnets always on the same horizontal line. At the moment when the distance between the magnets is $d_1 = 4$ cm and the hanged magnet has moved $x_1 = 1$ cm from initial position, balance is lost and the magnets pull together. By making the assumption that the pulling force between the magnets F_m depends on the distance d according to the relation $F_m \propto d^{-n}$, find the value of the exponent n.

4. SUPERBALLS (5 points) $n+1$ elastic balls are dropped so that they are exactly above each other, with a very small gap between each. Bottom ball has a mass of m_0, the one above has a mass of fm_0, next f^2m_0 and so on, until the topmost ball with mass $f^n m_0$, where $f < 1$. At the moment when the bottom ball touches the ground, all the balls are moving with the speed v.

i) *(1 point)* After the collision between the two bottommost balls, what is the speed v_1 of the second ball from the bottom?

ii) *(3 points)* What is the speed of the topmost ball v_n after all collisions?

iii) *(1 point)* How many times higher would that ball fly compared to the initial drop height h? Take $f = 0.5$ and $n = 10$.

It maybe useful that that sequence $a_0 = 1$, $a_{k+1} = a_k\alpha + \beta$ has a general term $a_n = \alpha^n + \beta\frac{\alpha^n - 1}{\alpha - 1}$, where α and β are constants.

5. PLANCK'S CONSTANT (8 points) In a simplistic model, light emitting diodes can be considered to only pass current when lit, and then they have a constant voltage drop $V_t = \frac{E}{e}$ across them. $E = hf$ is the energy of the light quanta emitted and $e = 1.60 \times 10^{-19}$ C is the elementary charge. Speed of light in vacuum $c = 3.00 \times 10^8$ m/s.

You have a assorted light emitting diodes numbered 1–6, each with a $R = 680\,\Omega$ series resistor. From the datasheets it is known for the peak wavelengths of the diodes to be 940 nm, 620 nm, 590 nm, 525 nm, 470 nm, and 450 nm.

i) *(2 points)* Find out which wavelength corresponds to which diode.

ii) *(4 points)* Measure the Planck's constant h that corresponds to our simplistic model. This does not have to correspond to real Planck's constant.

iii) *(2 points)* Estimate the uncertainty.

Equipment: voltage source (batteries) with an unknown voltage, ammeter, assorted light emitting diodes with series resistor. Take care not to short the battery with the ammeter. You may ignore the internal resistances of the batteries and the ammeter.

Estonian-Finnish Olympiad 2014

6. RUNNING ON ICE (4 points) A boy is running on a large field of ice with velocity $v = 5\,\mathrm{m/s}$ toward the north. The coefficient of friction between his feet and the ice is $\mu = 0.1$. Assume as a simplification that the reaction force between the boy and the ice stays constant (in reality it varies with every push, but the assumption is justified by the fact that the value averaged over one step stays constant).

i) *(2 points)* What is the minimum time necessary for him to change his moving direction to point towards the east so that the final speed is also $v = 5\,\mathrm{m/s}$?

ii) *(2 points)* What is the shape of the optimal trajectory called?

7. SPIN SYSTEM (8 points) Let us consider a system of N independent magnetic dipoles (spins) in a magnetic field B and temperature T. Our goal is to determine some properties of this system by using statistical physics. It is known that the energy of a single spin is $E = \epsilon m$, where $m = \pm\frac{1}{2}$ and $\epsilon = \alpha B$.

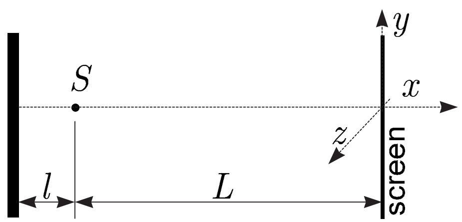
i) *(2 points)* What is the probability $p_\uparrow$ for a spin to be in exited state, i.e. have positive energy?

ii) *(2 points)* What is the average value of the total energy E_s of the spin system as a function of B and T?

iii) *(2 points)* Using high temperature approximation $T \gg \frac{\alpha B m}{k}$, simplify the expression of E_s.

iv) *(2 points)* Using high temperature approximation $T \gg \frac{\alpha B m}{k}$, find the heat capacity C of the spin system.

8. MIRROR INTERFERENCE (5 points) A point source S emits coherent light of wavelength λ isotropically in all directions; thus, the wavefronts are concentric spheres. The waves reflect from a dielectric surface placed at a distance $l = N\lambda$ (where N is a large integer) from the point source, and the interference pattern is observed on a screen which is placed to a distance $L \gg l$ from the point source (see figure).



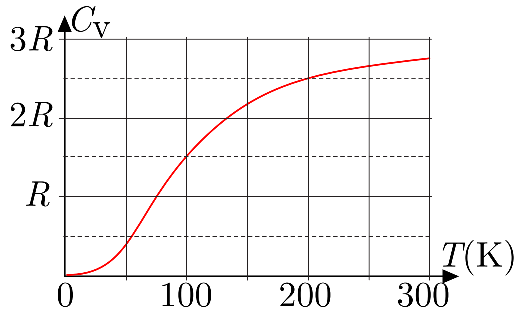
In what follows we use the x, y, and z coordinates as defined in the figure. The screen is parallel to the mirror and lies in the $y-z$-plane.

i) *(2 points)* At which values of the y-coordinate (for $z = 0$) are the interference maxima observed on the screen? You may assume that $y \ll L$.

ii) *(1 point)* Sketch the shape of few smallest-sized interference maxima on the screen (in $y-z$-plane).

iii) *(2 points)* Now the flat screen is replaced with a spherical screen of radius L, centred around the point source. How many interference maxima can be observed?

9. THERMAL ACCELERATION (9 points) Consider a cube of side length $a = 1\,\mathrm{cm}$, made of aluminium (density $\rho = 2.7\,\mathrm{g/cm^3}$, molar mass $M_A = 23\,\mathrm{g/mol}$). The heat capacitance of one mole of aluminium is given as a function of temperature in the graph below. The speed of light $c = 3\times10^8\,\mathrm{m/s}$, universal gas constant $R = 8.31\,\mathrm{J/(kg\cdot K)}$. The initial temperature of the cube is $T_0 = 300\,\mathrm{K}$.



i) *(1 point)* What is the total heat energy of such a cube at the initial temperature T_0?

ii) *(3 points)* Now, the cube has 5 faces painted in white (reflects all relevant wavelengths) and one face painted in black (absorbs all these waves). The cube is surrounded by vacuum at a very low temperature (near absolute zero); there is no gravity field. Initially, the cube is at rest; as it cools down due to heat radiation, it starts slowly moving. Estimate its terminal speed v_1.

iii) *(2 points)* At very low temperatures, the heat capacitance of aluminium is proportional to T^3, where T is its temperature. Which functional dependance $f(t)$ describes the temperature as a function of time [$T = A\cdot f(Bt)$, where A and B are constants] for such very low temperatures under the assumptions of the previous question?

iv) *(3 points)* Now, the cube has 5 faces covered with a thermal insulation layer (you may neglect heat transfer through these faces). One face is left uncovered. The cube is surrounded by hydrogen atmosphere at a very low temperature (molar mass of hydrogen molecules $M_H = 2\,\mathrm{g/mol}$). The cube starts cooling down due to heat transfer to the surrounding gas; you may neglect the heat radiation. Initially, the cube is at rest; as it cools down, it starts slowly moving. Estimate the order of magnitude of its terminal speed v_2. Assume that the surrounding gas is sparse, so that the mean free path of the molecules is much larger than a. Assume that $v_2 \ll c_s$ where c_s is the speed sound in the atmosphere surrounding the cube.

10. YOUNG'S MODULUS OF RUBBER (12 points) The linear Hooke's law for a rope made from an elastic material is supposed to held for small relative deformations $\varepsilon = x/L$ (which is also called "strain"), where L is the undeformed length of the rope, and x is the deformation. Once ε becomes too large, the force-deformation relationship $F = kx$ is no longer linear; what is "too large" depends on the material. For very elastic materials which can reach relative deformations considerably large than one, it may happen that the linear Hooke's law with a constant stiffness k fails, but if we take into account the change of the cross-sectional area S of the rope with $k = ES/L$, where E is the Young's modulus of the elastic material, such a non-linear Hooke's law remains valid. In that case we can say that there is still a linear stress-strain relationship $\sigma = E\cdot\varepsilon$, where the stress $\sigma = F/S$.

i) *(7 points)* Measure the relationship between the stress σ and strain ε in a rubber string and plot it.

ii) *(5 points)* From your plot determine the Young's modulus E with its uncertainty, and the maximum strain ε_m until which it applies.

Note: the diameter of the thread is to be measured using the diffraction of laser light.

Equipment: rubber thread, stand, measuring tape, 15 hex nuts with a known mass, a plastic bag for hanging a set of nuts to the thread, a green laser ($\lambda = 532\,\mathrm{nm}$), a screen.

WARNING: AVOID LOOKING INTO A LASER BEAM, THIS MAY DAMAGE YOUR EYES!