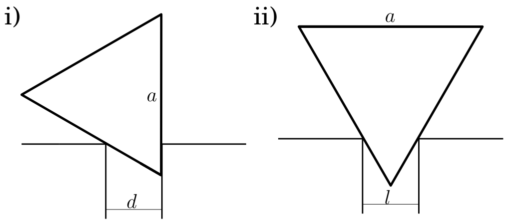
Estonian-Finnish Olympiad 2013

1. PRISM (8 points)

i) *(4 points)* A right prism that has an equilateral triangular base with length a is placed in a horizontal slit between two tables, so that one of the side faces is vertical. How small can the width d of the slit be made before the prism falls out of the slit? There is no friction between the prism and the tables and the prism is made of a homogeneous material. The edges of the slit are parallel.

ii) *(4 points)* Now the prism is placed in the slit so that one of its side faces is horizontal. How small can the width l of the slit be made before that position becomes unstable?



2. CELLPHONE CAMERA (6 points)

A photographer focussed his camera to distance L and took a photo. On the photo, all farther objects (up to infinity) turned also out to be sharp. Additionally, all closer objects down to distance s were sharp.

i) *(4 points)* What is the minimum possible L?

ii) *(2 points)* Find the corresponding s.

Background. We consider the image of a pointlike object to be sharp if its image is smaller than one pixel on the sensor. Otherwise the image is blurry. The lens of the camera may be viewed as a convex lens. The camera is focussed by changing the distance between the sensor and the lens.

Parameters. Calculate the answer for a cellphone made by a well-known company. The focal length of its camera $f = 4.3\,\mathrm{mm}$ and the diameter of the lens $D = 1.8\,\mathrm{mm}$. The sensor is $w = 4.6\,\mathrm{mm}$ wide corresponding to $N = 3264$ pixels.

3. MISSION TO MARS (7 points)

A crew of astronauts is going to be sent to explore the polar region of Mars and search for buried water ice. Their spaceship will travel from Earth to Mars along an elliptic transfer orbit tangential to the orbits of both planets. Despite its shortcomings, this orbit is commonly used in space travel due to its relatively good fuel economy. Future manned missions to Mars are very likely based on this kind of transfer. In this problem you will examine some aspects of this orbit.

The mean orbital radius of Mars is $R_a = 1.52\,\mathrm{AU}$. The mean orbital radius of Earth is $R_g = 1\,\mathrm{AU} = 149\,600\,000\,\mathrm{km}$. Mars has a mean radius of $r_a = 3397\,\mathrm{km}$ and surface gravity $g_a = 3.71\,\mathrm{m/s^2}$. Earth has a mean radius of $r_g = 6371\,\mathrm{km}$.

i) *(1 point)* Find the orbital period T_a of Mars, i.e. find the length of Martian “year” in Earth years.

ii) *(1.5 points)* How long (t_t) does a one-way trip to Mars take?

iii) *(1.5 points)* The spaceship is put into this orbit by using a powerful rocket. It is more efficient to burn fuel as a short burst when the spaceship is still near Earth. How much additional speed (Δv_1) does the booster have to be able to give to the spaceship to enter the transfer orbit, starting from the north pole? Neglect the air resistance.

iv) *(1.5 points)* Estimate the Δv_2 needed to enter a circular orbit close to Mars.

v) *(1.5 points)* What is the minimal duration of the trip to Mars and back?

4. MAGNETIC DIPOLES (7 points)

Let us consider the following model for a magnetic dipole. Some wire with no resistance has been bent into the shape of a square with side length a. At some point on the wire is a small ideal current source that keeps current I flowing in the circuit in all situations. The magnetic moment m of a planar circuit is given by the relation $m = IA$, with vector $\vec{m}$ pointing in the normal direction of the circuit according to the right hand rule (A is the area bounded by the circuit).

i) *(3 points)* The dipole is placed inside a homogeneous magnetic field $\vec{B}$, so that the angle between $\vec{m}$ and $\vec{B}$ is θ. Find the angles θ_s and θ_u that correspond to stable and unstable equilibria, respectively. Calculate the amount of work (w) needed to rotate the dipole from θ_s to θ_u. Give your answer in terms of m and B.

We can use this model to calculate the magnetic properties of materials containing unpaired electrons that have negligibly weak interactions with one another. Let us consider a sample of material with n such unpaired electrons per unit volume, placed inside a homogeneous magnetic field $\vec{B}$. Due to spin, each unpaired electron acts as a small magnetic dipole. However, owing to the quantum nature the electron, the projection of its magnetic moment along $\vec{B}$ can only be μ_B or $-\mu_B$ (μ_B is called the Bohr magneton).

ii) *(4 points)* Calculate M, the magnetic moment per unit volume of the sample, if the temperature of the material is T and the external magnetic field is B.

5. FRICTION OF A STRING (8 points)

Measure the dynamic coefficient of friction μ_1 between the ballpoint pen and the string. Estimate the uncertainty. It might help that the dynamic coefficient of friction between the pencil and the same string was measured beforehand and $\mu_2 = 0.20 \pm 0.01$ was obtained.

Equipment: dynamometer, string, ballpoint pen, pencil and weight.

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6. SPHERE AND CYLINDER (7 points) A sphere and a cylinder are lying on an inclined surface with inclination angle α. Both have mass m and radius r. The bodies are released from equal initial heights H. The moments of inertia of the sphere and the cylinder are, respectively, $I_{\text{sph}} = \frac{2}{5}mr^2$ and $I_{\text{cyl}} = \frac{1}{2}mr^2$. The coefficient of friction between the surface and the bodies is μ.

i) *(2 points)* Which of the bodies comes down faster? What was the relative lag of the slower body $\gamma = (t_2 - t_1)/t_1$? The times t_1 and t_2, respectively, denote the traveling times of the faster or the slower body. Assume that the rolling occurs without slipping.

ii) *(2.5 points)* Find the minimal angle of inclination α_0 for which the cylinder starts to slide in addition to rolling.

iii) *(2.5 points)* If $\alpha \to 90°$, the bodies obviously lose contact with the surface and fall down in free fall with equal times. What is the minimal angle of inclination α_m, for which both the sphere and the cylinder come down with equal times?

7. BURNING WITH A LENS (7 points) Sunrays are focused with a lens of diameter $d = 10\,\text{cm}$ and focal length of $f = 7\,\text{cm}$ to a black thin plate. Behind the plate is a mirror. Angular diameter of the Sun is $\alpha = 32'$ and its intensity on the surface of the Earth is $I = 1000\,\text{W/m}^2$, Stefan-Boltzmann constant $\sigma = 5.670 \times 10^{-8}\,\text{W/(m}^2\text{K}^4)$.

i) *(4 points)* Find the temperature of the heated point of the plate.

ii) *(3 points)* Using thermodynamic arguments, estimate the maximal diameter of the lens for which this model can be used.

8. ZENER DIODE (7 points) An inductance L and a capacitor C are connected in series with a switch. Initially the switch is open and the capacitor is given a charge q_0. Now the switch is closed.

i) *(1 point)* What are the charge q on the capacitor and the current I in the circuit as functions of time? Draw the phase diagram of the system — the evolution of the system on a $I - q$ graph — and note the curve's parameters. Note the direction of the system's evolution with arrow(s).

A *Zener diode* is a non-linear circuit element that acts as a bi-directional diode: it allows the current to flow in the positive direction when a forward voltage on it exceeds a certain threshold value, but it also allows a current to flow in the opposite direction when exposed to sufficiently large negative voltage. Normally the two voltage scales are quite different, but for our purposes we will take a Zener diode with the following volt-ampere characteristics: for forward currents, the voltage on the diode is V_d, for reverse currents, the voltage on the diode is $-V_d$, for zero current the voltage on the diode is $-V_d < V < V_d$.

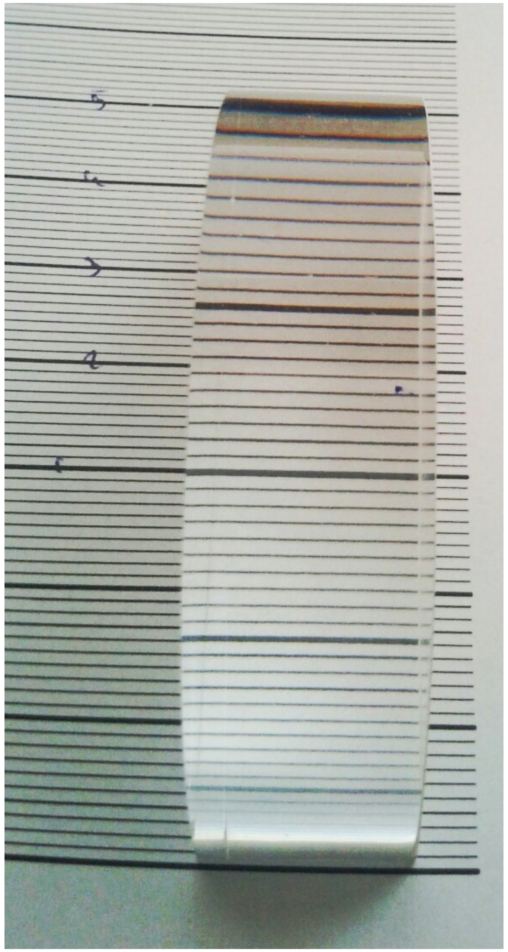
Now we connect the inductance L, the capacitor C all in series with a switch and a Zener diode. The switch is initially open. The capacitor is again given the charge $q_0 > CV_d$ and the switch is then closed.

ii) *(2 points)* Make a drawing of the phase diagram for the system. Note the direction of the system's evolution with arrow(s).

iii) *(2 points)* Does the evolution of the system only necessarily stop for $q = 0$? Find the range of values of q on the capacitor for which the evolution of the system will necessarily come to a halt.

iv) *(2 points)* Find the decrease Δq in the maximum positive value of the capacitor's charge q after one full oscillation. How long does it take before oscillation halts?

9. GLASS CYLINDER (7 points) In the following figure, there is a half-cylinder, made of glass and put on a paper with stripes (the inter-stripe distance is everywhere the same). Find the coefficient of refraction of the glass.



10. RESISTIVE HEATING (8 points) Measure the resistor. You are not asked to estimate the uncertainty.

Equipment: resistor, voltage source (batteries), ammeter, calorimeter, thermometer, stopwatch.

The calorimeter has $V = 0.80\,\text{dl}$ of water and $m_a = 27\,\text{g}$ of aluminum, specific heat capacity of water $c_w = 4.2\,\text{J/(K}\cdot\text{g)}$ and of aluminum $c_a = 0.90\,\text{J/(K}\cdot\text{g)}$. Internal resistance of the batteries will vary. Your set of batteries may become depleted, spares are available.