Estonian-Finnish Olympiad - 2012

Problem 1. Asteroid *(14 points)*

Consider a hypothetical asteroid of mass m_a and radius r_a, which moves along an elliptical orbit around the Sun (of mass M_s) in the same direction as Earth. Let us assume that the Earth's orbit is a circle of radius R_e (neglecting thus its eccentricity) and that the two orbits lay in the same plane. The asteroid's shortest distance to the Sun (at its perihelion) $R_{\min} = \frac{1}{2}R_e$ and the longest distance (at the apohelion) $R_{\max} = 1.51R_e \approx 1.5R_e$ (you may use the approximate value to simplify your calculations). The orbital velocity of the Earth $v_0 = 30\,\mathrm{km/s}$. You can also use the following numerical values: the radius of the Earth $r_e = 6400\,\mathrm{km}$, free fall acceleration at the Earth's surface $g = 9.81\,\mathrm{m/s^2}$, angular diameter of the Sun as seen from the Earth $\alpha = 0.5°$, duration of one year $T_0 = 365$ days, the temperature of the Sun's surface $T_s = 6000\,\mathrm{K}$, free fall acceleration at the Sun's surface $g_s = 275\,\mathrm{m/s^2}$, Stefan Boltzmann constant $\sigma = 5.6704 \times 10^{-8}\,\mathrm{kg \cdot s^{-3} \cdot K^{-4}}$, the speed of light $c = 3 \times 10^8\,\mathrm{m/s}$. The asteroid is of a spherical shape, its radius $r_a = 10\,\mathrm{m}$ and mass $m_a = 1 \times 10^7\,\mathrm{kg}$; both the Sun and the asteroid can be considered as perfectly black bodies.

Part A. Collision with Earth *(5 points)*

i. (2 pts) Suppose that the asteroid will collide with the Earth and is already very close, at a distance $l \ll R_e$ from the Earth's surface; what is the velocity of the asteroid with respect to the Earth assuming that (a) $l \gg r_e$; (b) $l \ll r_e$.

ii. (2 pts) Impact parameter b is defined in the Earth's reference frame as the distance between the Earth and a line, tangent to the asteroid's trajectory at a point which is far enough (at a distance l, $r_e \ll l \ll R_e$). Determine the maximal value of the impact parameter $b_{\max}$ for which the asteroid will still collide with the Earth.

iii. (1 pt) According to calculations, the asteroid is going to hit the Earth centrally after $N = 10$ orbital periods. In order to avert the collision, the period of the asteroid needs to be changed; by how many seconds? (Assume simplifyingly that the intersection point of the two orbits remains at rest.)

Part B. Changing the solar pull *(9 points)*

Theoretically, it is possible to change the asteroid's period by making use of the pressure of the solar radiation. Let us study, how realistic is such a project. The Sun pulls the asteroid with the gravitational force equal to $F_0 = GM_s m_a/R^2$, where G is the gravitational constant and R is the distance between the Sun and the asteroid at the given moment of time. Let us denote $GM_s = \gamma_0$, so that

$$F = \gamma_0 m_a/R^2.$$

Suppose that when the asteroid is at its perihelion, the constant γ_0 is decreased instantaneously, down to a new value γ_1, which remains constant during the subsequent motion.

i. (2 pts) Find the new apohelion distance $R'_{\max}$ of the asteroid; express it in terms of $\kappa = (\gamma_0 - \gamma_1)/\gamma_0$.

ii. (2 pts) Find the change of the asteroid's orbital period assuming that $\kappa \ll 1$ (express it in terms of κ).

iii. (4 pts) Suppose that at its perihelion, the asteroid is coated with a perfectly retro-reflective paint (which directs all the incident light directly back, towards the source). Such a painting will result in a change of the effective pull of the asteroid towards the Sun; find the respective value of κ (provide also a numerical estimate).

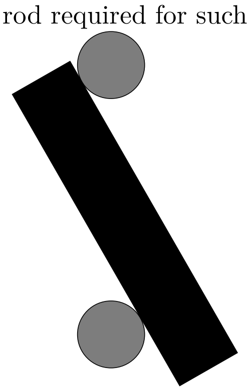
iv. (1 pt) Estimate, is it realistic to avert the collision with this asteroid using the retroreflective paint.

Problem 2. Thermodynamic cycle *(5 points)*

Calculate the thermal efficiency of an ideal-gas cycle consisting of two isotherms at temperatures T_1 and T_2, and two isochores joining them. (An isochore is a constant-volume process.) The engine is constructed so that the heat released during the cooling isochore is used for feeding the heating isochore.

Problem 3. Bars and rod *(5 points)*

Two cylindrical horizontal bars are fixed one above the other; the distance between the axes of the bars is $4d$, where d is the diameter of the rod. Between the bars, a cylindrical rod of diameter d is placed as shown in Figure (this is a vertical cross-section of the system). The coefficient of friction between the rod and the bars is $\mu = \frac{1}{2}$. If the rod is long enough, it will remain in equilibrium in such a position. What is the minimal length L of the rod required for such an equilibrium?



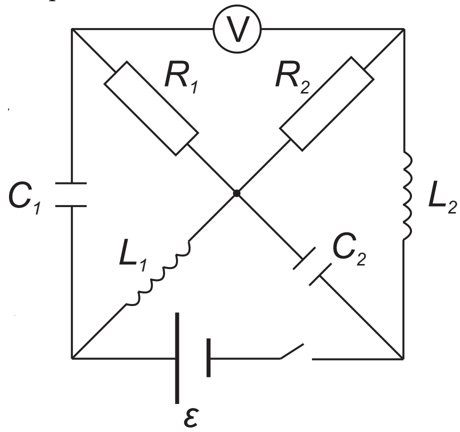
Problem 4. RLC-circuit *(5 points)*

For the circuit shown in Figure, $R_1 = 3R$, $R_2 = R$, $C_1 = C_2 = C$, and $L_1 = L_2 = L$. The electromotive force of the battery is $\mathcal{E}$. Initially the switch is closed and the system is operating in a stationary regime.

i. (1 pt) Find the reading of the voltmeter in the stationary regime.

ii. (2 pts) Now, the switch is opened. Find the reading of the voltmeter immediately after the opening.

iii. (2 pts) Find the total amount of heat which will be dissipated on each of the resistors after opening the switch, and until a new equilibrium state is achieved.



Problem 5. Diffraction grating *(7 points)*

Determine the pitch (the distance between the neighbouring lines) of the reflecting diffraction grating; estimate the uncertainty of the result. Equipment: a reflecting diffraction grating, green laser ($\lambda = 532\,\mathrm{nm}$), ruler, a stand for holding the laser.

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Problem 6. Uranium decay *(7 points)*

Natural uranium consists of 99.3% U^{238}, and in this problem we can ignore the presence of other isotopes. U^{238} decays according to the table at the bottom of the page, where an isotope decays into the isotope indicated in its neighbouring cell (to the right), the decay energy is given in the cell below in megaelectronvolts (MeV), $1\,\mathrm{MeV} = 1.6 \times 10^{-13}\,\mathrm{J}$, and the last row indicates the logarithm of the respective half-life in seconds, $\log_{10} \tau_{\frac{1}{2}}$ (so that 17.15 below U^{238} means that the half-life of U^{238} is $10^{17.15}\,\mathrm{s} \approx 1.41 \times 10^{17}\,\mathrm{s}$.

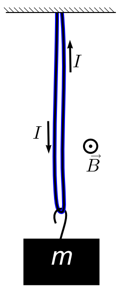
You may also use the value of the Avogadro number $N_A = 6.02 \times 10^{23}\,\mathrm{mol}^{-1}$, and the following physical properties of uranium: melting point $T_0 = 1408\,\mathrm{K}$, density $\rho = 1.89 \times 10^4\,\mathrm{kg/m^3}$, thermal conductivity $\kappa = 27.5\,\mathrm{W/m \cdot K}$, molar mass $\mu = 0.238\,\mathrm{kg/mol}$. It can be assumed that the radon which appears in the chain of nuclear decays does not escape the bulk of uranium. *Remark:* thermal conductivity is the coefficient of proportionality between the heat flux density ($\mathrm{W/m^2}$) and dT/dx (temperature drop per unit distance).

i. (2 pts) Assuming that the only source of natural U^{234} is the decay chain of U^{238}, what is the percentage of U^{234} in the natural uranium ore?

ii. (2 pts) Determine the heat production volume rate of natural uranium w (in watts per cubic meter) due to nuclear decay.

iii. (3 pts) If the radius of a uranium ball is large enough, it will melt inside. How large radius R_0 is needed for such a melting, if ambient temperature is $T_a = 300\,\mathrm{K}$?

Problem 7. Lifting by current *(7 points)*



Consider a loop of freely deformable conducting wire with insulation of length $2l$, the two ends of which are fixed (permanently) to the ceiling. A load of mass m is fixed to the middle of the wire (the mass of the wire is negligible). There is also a horizontal magnetic field of induction B; free fall acceleration is g. A current I is lead through the wire. Neglect the field induced by the wires.

i. (2 pts) Sketch the shape of the wire.

ii. (2 pts) What is the maximal height by which the load can be lifted in such a way (increasing the current, if necessary)?

iii. (2 pts) Write an equation from which it is possible to determine the lifting height Δh.

iv. (1 pt) How large current I_0 is needed to lift the load by $\Delta h_0 = l(1 - \frac{3}{\pi})$?

Problem 8. Elastic collision *(7 points)*

Consider a perfectly elastic collision of two balls, one of which has mass M and is moving with velocity v. The other has mass $m \leq M$ and stays initially at rest. The collision is not necessarily central. The surface of the balls is slippery, so the balls will not rotate.

i. (1 pt) What are the momenta of the balls before the collision in the frame of reference where the centre of mass of the whole system stays at rest?

ii. (3 pts) What are the momenta of the balls (by moduli) after the collision in the mass centre's frame of reference?

iii. (3 pts) What is the maximal angle α by which the trajectory of the initially moving ball can be inclined as a result of the collision?

Problem 9. Power lines *(7 points)*

i. (2 pts) Consider a rope of linear density σ (mass per unit length), which is stretched so as to maintain a mechanical tension T in the rope. Show that along such a rope, perturbations (eg. waves) will travel with the speed $k\sqrt{T/\sigma}$, and find the coefficient k.

ii. (2 pts) Now, consider a power line between two poles, the distance between of which is L; the linear density of the wire is σ, and the middle point of the wire hangs below the horizontal level at which the wire is fixed to the poles by a distance d. Find the tension T in the wires assuming that $d \ll L$. *Hint:* make use of the torque balance for a half of the wire.

iii. (3 pts) Find the lowest frequency f_0 of free oscillations of such a power line (the amplitude of the oscillations is so small that the tension in the wire remains essentially constant).

Problem 10. Black box *(8 points)*

Determine the electric circuit inside the black box. It is known that apart from the wires and two resistors, the electric circuit includes four components. Equipment: black box with four output leads, a piece of wire.

U^{238}	Th^{234}	Pa^{234}	U^{234}	Th^{230}	Ra^{226}	Rn^{222}	Po^{218}	Pb^{214}	Bi^{214}	Po^{214}	Pb^{210}	Bi^{210}	Po^{210}	Pb^{206}
4.27	0.27	2.27	4.86	4.77	4.87	5.59	6.12	1.02	3.27	7.88	0.06	1.43	5.41	–
17.15	6.32	4.38	12.89	12.38	10.70	5.52	2.27	3.21	3.08	-3.78	8.85	5.64	7.08	stable