Estonian-Finnish Olympiad - 2010

1. Charges in E ***(8 points)*** Two particles (the blue and the red) of mass m are connected with a spring, the stress-free length of which is L and stiffness k; the blue carries charge q ($q > 0$) and the red is chargeless. In the region $x > 0$, there is an homogeneous electric field E, antiparallel to the x-axis; In the region $x < 0$, there is no electric field. Initially, the "dumbbell" of charges moves in region $x < 0$ with velocity v, parallel to the x-axis; the dumbbell's axis is also parallel to the x-axis and the spring is stress-free. It is known that after a while, the dumbbell moves in the region $x < 0$ with velocity $-v$, and that the red particle never enters the region $x > 0$. Also, the spring's length achieves minimum only once.

i) (2.5 pt) How long time τ does the blue particle spend in the region $x > 0$?

The process takes place exactly as described, if one equality and one inequality are satisfied for the quantities m, v, k, q, E and L.

ii) (3 pt) Which equality must be satisfied?

iii) (2.5 pt) Which inequality must be satisfied?

2. Thermos bottle ***(6 points)*** In order to study the thermal properties of a thermos bottle, let us model it as two concentric spherical vessels, with radii $R_1 = 7\,\mathrm{cm}$ and $R_2 = 10\,\mathrm{cm}$. The gap between the walls of the vessels contains vacuum (hence, the heat conductivity can be neglected).

i) (3,5 pt) Find the radiative heat flux (i.e. transmitted heat per unit time) between the walls of the bottle, assuming that the ambient temperature is $T_2 = 293\,\mathrm{K}$ and the inner sphere is filled with liquid nitrogen at the boiling temperature $T_1 = 77\,\mathrm{K}$. The emissivities of all the surfaces are equal to that of stainless steel: $\varepsilon = 0.1$. *Remark:* The emitted heat flux per unit area is given Stefan-Boltzmann's law $P = \varepsilon\sigma T^4$, where $\sigma = 5.67 \times 10^{-8}\,\mathrm{W/m^2K^4}$ (assuming that ε is independent of the wavelength.

ii) (2,5 pt) Estimate, how long time does it take for a full evaporization of the liquid nitrogen (the vapor escapes through an over pressure valve). For the liquid nitrogen, density $\rho = 810\,\mathrm{g/l}$ and latent heat for vaporization $\lambda = 5.580\,\mathrm{kJ/mol}$). *NB! If you were unable to find P (for question i), express the evaporization time symbolically (i.e. using the symbol P).*

3. Tyrannosaur (T. Rex) ***(6 points)*** Paleontologists have discovered tracks of a tyrannosaur where the footprints of the same leg are $A = 4.0$ meters apart. They have also recovered a piece of a tyrannosaur leg bone that has bone cross-sectionalo area $N = 10000$ times that of a chicken (which the tyrannosaur is related to).

i) (3 pt) Knowing that the chicken leg is approximately $l = 15$ cm tall, estimate the length of a tyrannosaur leg L. You may assume that the length of a leg scales as the length of the whole animal, and that the bone stress (force per area) is the same for both animals. Is your result consistent with the step length A?

ii) (3 pt) Estimate the natural walking speed of the tyrannosaur by approximating the walking motion of a leg with a freely oscillating pendulum motion. State clearly all the assumptions you make.

4. Ball ***(6 points)*** Massive spherical ball has a mass $M = 100$ kg; an attempt is made to roll the ball upwards, along a vertical wall, by applying a force F to some point P on the ball. The coefficient of friction between the wall and the ball is $\mu = 0.7$.

i) (5 pt) What is the minimal force F_{min} required to achieve this goal?

ii) (1 pt) On a side view of the ball and the wall, construct geometrically the point P, where the force has to be applied to, together with the direction of the applied force.

5. Elastic thread ***(10 points)*** Equipment: ruler, tape, an elastic thread, a wooden rod, marker, a known weight.

The purpose of this problem is to study the elastic properties of an elastic thread for large relative deformations $\varepsilon = (l - l_0)/l_0$, where l_0 and l are the lengths in initial and stretched states, respectively. If the Hook's law were valid, the ratio F/ε of the elastic force F and ε would be constant: $F/\varepsilon = SE$, where S is the cross-section area of the thread and E — the Young modulus of the thread material.

i) Collect the data needed to plot the ratio F/ε as a function ε, up to $\varepsilon \approx 4$. Plot the appropriate graph, and indicate the uncertainties.

ii) By making assumption that the Young modulus $E = F/S\varepsilon$ remains constant, study, how does the volume of the thread depend on ε. Plot the appropriate graph.

6. Charges in B *(5 points)* There is an homogeneous, parallel to the z-axis magnetic field of inductance B in region $x > 0$. In region $x < 0$, there is no field. There are two particles of mass m and charge q. Initially, the particles have coordinates $y = z = 0$, and respectively $x = -L_0$ and $x = -2L_0$ (with $L_0 > 0$). Initial velocity of both particles is v, along the x-axis, towards the magnetic field. Neglect the electrostatic repulsion force of the two charges.

i) (1,5 pt) Sketch the trajectory of the first particle, and the dependance of its y-coordinate on time.

ii) (3,5 pt) Sketch the distance L between the particles as a function of time t, assuming that $\pi mv/Bq > L$. What is the minimal distance $L_{\min}$?

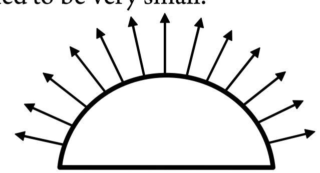
7. Satellite *(5 points)*

i) (3 pt) A large ball of mass m_1 is kept at the height h from the floor (so that the center of the ball is at the height $h + d/2$, where d is its diameter). A small ball of mass m_2 is placed upon the large one, and the system is released (so that it starts falling). To which height (from the floor) will the small ball rise, assuming that the collision between the lower ball and ground, and the collision between the balls are absolutely elastic, and $m_1 \gg m_2$?

ii) (2 pt) Consider the following satellite launching project. There are N absolutely elastic balls of masses $m_1 \gg m_2 \gg \ldots \gg m_N$: the first ball (the heaviest) is the lowest; the second ball is placed on top of the first; the third — on top of the second etc. The upmost ball is supposed to become a satellite, i.e. to obtain the velocity $v_N = 7.8\,\mathrm{km/s}$). The lowest ball is at the height $h = 1$ m from the floor, and the system is released. What should be the number of balls N? What should be the mass of the lowest ball, if $m_i/m_{i+1} = 10$, and the mass of the satellite $M_N = 1$ kg?

8. Sprinkler *(3 points)*

A sprinkler has a shape of hemisphere, which has small holes drilled into the spherical part of its surface. From these small holes, water flows out with velocity $v = 10\,\mathrm{m/s}$. Near the sprinkler, the water flow is distributed evenly over all the directions of the upper half-space. The sprinkler is installed at the ground level so that its axis is vertical. In what follows, the air resistance can be neglected, and the dimensions of the sprinkler can be assumed to be very small.

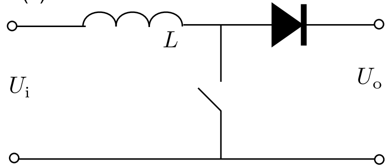


i) (1.5 pt) Find the surface area of the ground watered by the sprinkler.

ii) (1.5 pt) At which distance from the sprinkler is the watering intensity (mm/h) the highest?

9. Power supply *(6 points)*

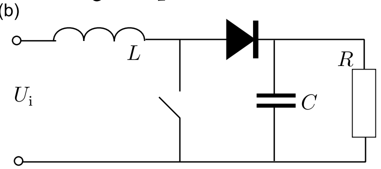
(a)



i) (2 pt) Consider the cirquit given in Fig (a), where the diode can be assumed to be ideal (i.e. having zero resistance for forward current and infinite resistance for reverse current. The key is switched on for a time τ_c and then switched off, again. The input and output voltages are during the whole process constant and equal to U_i and U_o, respectively ($2U_i < U_o$). Plot the graphs of input and output currents as functions of time.

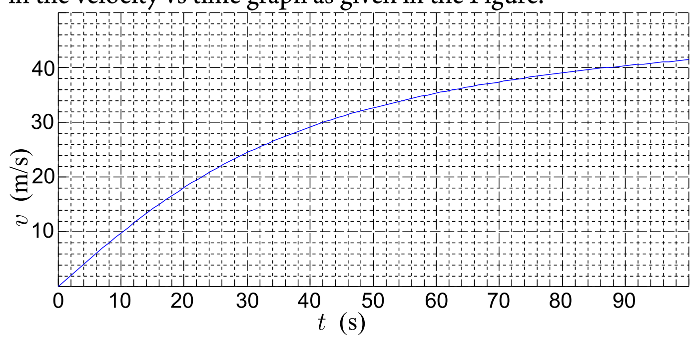
ii) (2 pt) Now, the key is switched on and off periodically; each time, the key is kept closed for time interval τ_c and open — also for τ_c. Find the average output current.

(b)



iii) (2 pt) Now, cirquit (a) is substituted by cirquit (b); the switch is switched on and off as in part ii. What will be the voltage on the load R, when a stationary working regime has been reached? You may assume that $\tau_c \ll RC$, i.e. the voltage variation on the load (and capacitor) is negligible during the whole period (i.e. the charge on the capacitor has no time to change significantly).

10. Ice-rally *(7 points)* The car accelerates on a slippery ground so that the wheels are always at the limit of slipping (e.g. via using an electronic traction control). Such an acceleration would result in the velocity vs time graph as given in the Figure.



i) (2 pt) What is the coefficient of friction, assuming a four-wheel drive?

Because of a manual gear change, there is time period of $\tau_1 = 0.5\,\mathrm{s}$, during which there is no driving force (so that the car decelerates due to air friction). Except for that period, the acceleration follows the law given by the graph. As a result, the terminal velocity $v_t = 40\,\mathrm{m/s}$ is reached $\tau_2 = 1.0\,\mathrm{s}$ later than it would have been reached, if there were no delay caused by the gear change. Upon reaching the terminal velocity, the car continues moving at constant speed. In your calculations, you can assume that the air friction was constant during the gear change period.

ii) (2,5 pt) At which speed the gear was changed?

iii) (2,5 pt) How many meters shorter distance will be covered during the first 100 seconds, as compared to ideal acceleration (i.e. without the delay due to the gear change)?

11. Black box *(10 points)* Equipment: a black box, multimeter, battery, timer (on the screen).

Determine the electrical scheme inside the black box, and the values of all the resistors inside it. Estimate the characteristics of other electrical components. It is known that apart from the wires, the total number of components is three.