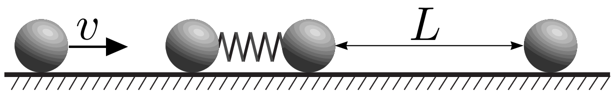
Eestonian-Finnish Olympiad - 2008

1. Dumbbell (6 points)

Two perfectly elastic identic balls of mass m are connected by a spring of stiffness k so that a dumbbell-like system is formed. That dumbbell lays at rest on a slippery horizontal surface (all the friction forces can be neglected). Third ball (identic to the ones making up the dumbbell) approaches coaxially the dumbbell from the left side with velocity v (see Figure). Fourth ball (identic to the other ones) lays coaxially rightwards to the dumbbell.

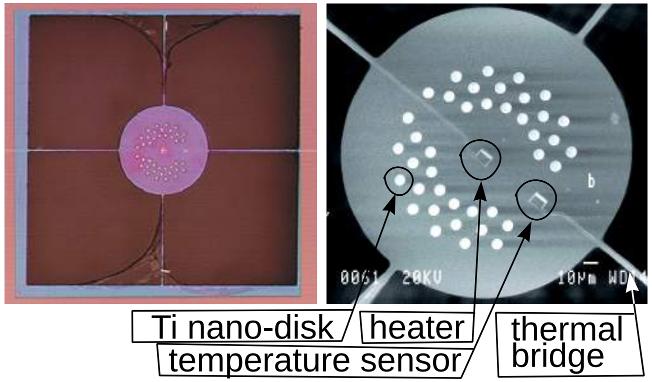
1) Find the velocity of the centre of mass of the dumbbell after being hit by the ball approaching from the left.

2) For which distances L between the dumbbell and the rightmost ball, the final velocity of the latter will be exactly the same, as the initial velocity v of the leftmost ball?



2. Microcalorimeter (9 points)

A microcalorimeter is a thin circular silicon nitride membrane, thermally isolated from the surroundings, except that it is thermally connected to the wafer by four thin and narrow thermal bridges (see Figure). The microcalorimeter is equipped with a small heater in the middle of the membrane and a similar structure on the edge of the membrane working as a thermometer. This micro calorimeter is used to study the thermal properties of nanoscale Ti disks (light tiny dots in Fig). The thermal power of the heater depends sinusoidally on time, $P = P_0 \cos(\omega t)$ (negative power implies a withdrawal of heat). The circular frequency ω is sufficiently low, so that for any moment of time t, the temperature of the microcalorimeter $T(t)$ can be considered constant across its entire surface, and the temperature profile along the thermal bridges can be considered linear. The wafer, to which the bridges are connected, is large and thick enough, so that its temperature T_0 can be considered to be constant all the time. Each of the four bridges have length L and cross sectional area of S; the thermal conductance of them is κ. Thermal conductance is defined as the heat flux (measured in Watts) per surface area, assuming that the temperature drop is $1\,^\circ\mathrm{C}$ per $1\,\mathrm{m}$. The heat capacity of the microcalorimeter (with Ti-disks) is C.



1) Find the thermal resistance R between the microcalorimeter and the wafer (i.e. the ratio of the temperature difference and heat flux).

For questions (ii) and (iii), use quantity R, without substituting it via the answer of question (i).

2) Write down the heat balance equation for the microcalorimeter and find the temperature of the microcalorimeter as a function of time $T(t)$ [you may seek it in the form $T = T_0 + \Delta T \sin(\omega t + \phi)$].

3) In order to study the thermal properties of the Ti-nanodisks, the amplitude of the sinusoidal oscillations of $T(t)$ should change by as large as possible value, as a response to a small change of C (which is caused by the Ti-disks). Find the optimal circular frequency ω_0.

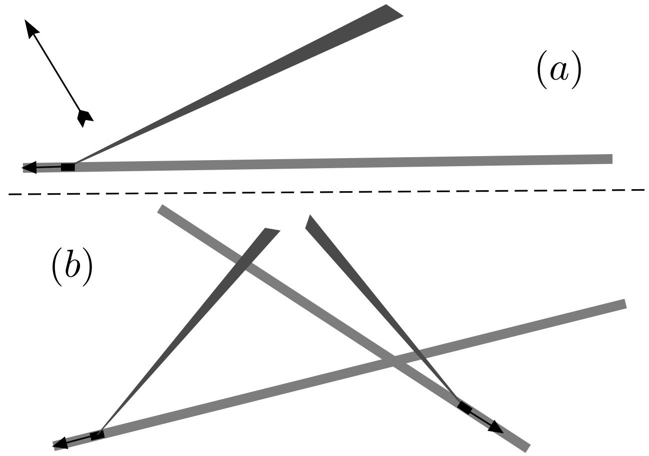
4) We have assumed that the temperature profile along the bridges is linear, i.e. their heat capacity can be neglected. For high frequencies $\omega \gtrsim \omega_c$, this is not the case. Estimate the critical frequency ω_c in terms of κ, l, specific heat c and density ρ of the bridge material.

3. Tractor (6 points)

Provided sketches (a) and (b) are made on the basis of satellite images, preserving proportions. They represent tractors, together their smoke trails. The tractors were moving along the roads in the direction indicated by the arrows. The velocity of all the tractors was $v_0 = 30\,\mathrm{km/h}$. For sketch (a), the direction of wind is indicated by another arrow.

1) Using the provided sketch, find the wind speed for case (a).

2) Using the provided sketch, find the wind speed for case (b).



4. Magnetic field (6 points)

Magnetic field with inductance B (parallel to the z-axis) fills the region $x^2 + y^2 < R^2$. Let us consider an electron of velocity $v = RBe/m$ (where e is its charge and m — its mass).

1) Sketch the trajectory of the particle, if initially, it moves along the line $y = 0$ towards the region filled with magnetic field.

2) How long time does the electron spend inside the magnetic field?

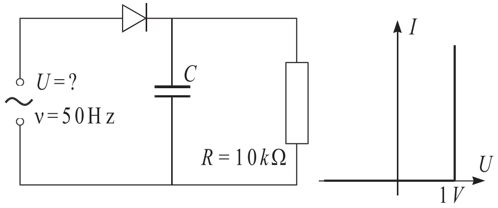
3) Now, let us consider the situation, when initially the electron moves along the line $y = a$ ($a < R$). Find the angle α, by which the electron is inclined after passing through magnetic field.

5. Ball (9 points)

Equipment: ruler, a glass ball, sheet of paper, marker. Find the coefficient of friction between the glass ball and the ruler. Estimate the uncertainty of you result.

6. Rectifier (8 points)

A voltage rectifier is made according to the circuit depicted in Figure. The load $R = 10\,\text{k}\Omega$ is fed with DC, equal to $I = 2\,\text{mA}$. In what follows we approximate the U-I characteristic of the diode with the curve depicted in Figure. The relative variation of the current at the load has to satisfy the condition $\Delta I/I < 1\%$.



1) Find the average power dissipation at the diode at the working regime of such a circuit.

2) Determine the amplitude of the AC voltage (with frequency $\nu = 50\,\text{Hz}$), which has to be applied at the input of the circuit.

3) Find the required capacitance C.

4) Find the average power dissipation at the diode during the first period (of AC input voltage) immediately following the application of AC voltage to the input of the circuit.

7. Fire (6 points)

There is wet wood burning in a fireplace on the ground. Seven meters above ground, the smoke is at a temperature of $t_7 = 40\,^\circ\text{C}$. Disregard the exchange of heat with the surrounding air and assume that the atmospheric pressure at the ground is constant in time and equal to $p_0 = 1000\,\text{hPa}$; the air temperature $t_0 = 20\,^\circ\text{C}$ is independent of height [1]. Assume that the smoke represents an ideal gas of a molar mass $\mu = 29\,\text{g/mol}$ (i.e. equal to the molar mass of the air), and of a molar specific heat at constant volume $C_V = 2.5R$; universal gas constant $R = 8.31\,\text{J/kg}\cdot\text{K}$. How high will the smoke column rise?

8. Electron (5 points)

Electron rests at the origin. At the moment of time $t = 0$, an electric field is switched in: its modulus is constant and equal to E_0, but its direction rotates with a constant circular velocity ω in the $x - y$ plane. At the moment $t = 0$, it is directed along the x-axes.

1) Find the average velocity of the electron over a long time interval for $t > 0$.

2) Sketch the trajectory of the electron and calculate the geometrical characteristics of it.

9. Asteroid (7 points)

It is believed that the impacts of the Earth with asteroids have played an important role in the history of Earth. In this problem, you are required to study such an impact. As an example, let us use the orbital data of the Apollo asteroid. Its perihelion is 0.65 AU, i.e. its closest distance from the Sun is $r_1 = \beta R$, where $\beta = 0.65$ and R denotes the radius of the Earth's orbit; its aphelion is 2.3 AU, i.e. its farthest distance from the Sun is $r_2 = \alpha R$, where $\alpha = 2.3$. In your calculations, you may use the orbital velocity of Earth, $v_0 = 30\,\text{km/h}$, Earth radius $R_0 = 6400\,\text{km}$, and the free fall acceleration at the Earth's surface, $g = 9.8\,\text{m/s}^2$. You may also use the formula $E = -GmM/2a$, expressing the total energy of a body of mass m, which moves along an elliptic orbit of longer semiaxis a, in the gravity field of a much heavier body of mass M. You may assume that the orbital planes of the Earth and of the asteroid coincide, and that they rotate in the same direction around the Sun.

1) Find the velocity v of the asteroid in the vicinity of the Earth, in the Sun's system of reference, neglecting the effect of Earth's attraction.

2) Find the radial and tangential components v_r and v_t of that velocity (i.e. the components, respectively parallel and perpendicular to the vector, drawn from the centre of the Sun to current position of the asteroid).

3) Find the same velocity components u_r and u_t in the Earth's system of reference.

4) Find the velocity of the asteroid w, immediately before entering the Earth's atmosphere (at the height $h = 100\,\text{km}$ from the Earth's surface).

10. Glass plate (10 points)

Equipment: Laser ($\lambda = 650\,\text{nm}$), thin glass plate, lens, ruler. NB! glass plate is fixed to a stand; avoid touching the glass itself (because its edges are sharp, and because it can break easily).

Find the thickness of the plate and estimate the uncertainty of the result. Draw the scheme of your experimental setup.

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[1] Actually, during day time, this is not the case: air temperature decreases with height. However, during evening and night, due to heat radiation, the lower layers of air cool more rapidly than upper layers, and it may easily happen that the temperature is roughly independent of height.