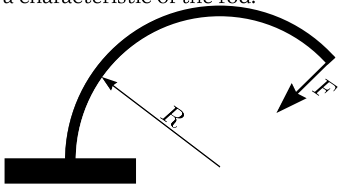
1. Wire (7 pts)

A conducting wire is formed of a cylindrical copper core with a diameter of $a = 2{,}5$ mm. The core is wrapped in a concentric, cylindrical aluminium coating, the total diameter of the wire being $b = 4$ mm. A current of $I = 2{,}4$ A flows through the wire. The specific resistivity of copper is $\rho_c = 0{,}0168 \cdot 10^{-6}\Omega{\cdot}\mathrm{m}$ and of aluminium, $\rho_a = 0{,}028 \cdot 10^{-6}\Omega{\cdot}\mathrm{m}$.
1) What are the current densities j in di erent parts of the wire (current density is defined as the current per cross-section area)?
2) What is the magnetic inductance B_1 at the distance $c = 1$ cm from the axis of the wire?
3) What is the magnetic inductance B_2 at the surface between the copper and aluminium?

Remark It may be useful to know the circulation theorem: $\int \vec{B}{\cdot}\vec{dl} = \mu_0 I$, where the integral is taken along a closed trajectory (loop) and I is the net current flowing though that loop; $\mu_0 = 4\pi \cdot 10^{-7}$ H·m^{-1}. This formula is completely analogous to the formula for the work done by a force along a trajectory: $\vec{F}$, $A = \int \vec{F} \cdot \vec{dl}$.

2. Pendulum (7 pts)

Consider an elastic rod, the mass and the compressibility of which (i.e. the length change) can be neglected in this problem. It can be assumed that if one end of the rod is firmly fixed, and a force F is applied to the other end of the rod, perpendicularly to the rod at the point of application, then the rod takes a form of a circle segment. The radius of that circle is inversely proportional to the force, $R = k/F$, where the factor k is a characteristic of the rod.

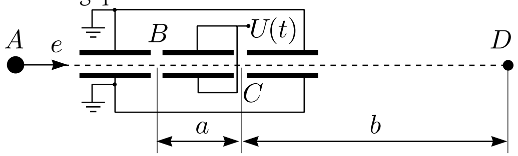


1) Let the rod be fixed vertically, at its bottom end, and a ball of mass m be attached to its upper end. Knowing the factor k, the length of the rod l, and the free fall acceleration g, find the period of small oscillations of the ball. In this question, you may assume that $gml \ll k$.
2) What is the maximal mass M of the ball, which can be stably held on such a vertical rod?

Remark: you may use approximate expressions $\sin x \approx x - x^3/6$ and $\cos x \approx 1 - x^2/2$ (for $x \ll 1$).

3. Temporal focusing (10 pts)

Suppose that at the point A, there is a source of thermal electrons (of negligible thermal energy), which are accelerated initially by the voltage $U_0 = 36$ V in horizontal direction (see figure). At the path of the electrons, there are two voltage gaps B and C of negligible size, at the distance a from each other. These gaps receive a voltage signal $U_C(t) = -U_B(t) \equiv U(t)$ from a waveform generator. We can assume that $|U(t)| \ll U_0$. The electrons starting at di erent moments of time are to be gathered together (focused) at the probe D, which is at distance b from the gap C. In order to analyze this setup, answer the following questions.



1) Assuming that $U(t) \equiv 0$, what is the time needed for the electrons to travel from the gap B to the probe D?
2) Find the same travel time assuming that the voltage $U(t) \equiv U \ll U_0$ is constant (your approximating expression should be a linear function of U).
3) What functional equation should be satisfied for the waveform $U(t)$ in order to ensure the focusing of all the electrons at the probe D. Solve this equation by assuming $a \ll b$ and $|U(t)| \ll U_0$.
4) The waveform generator yields a periodic signal of period T in such a way that the profile $U(t)$ is followed up to achieving some maximal value U_m; after that, the voltage drops immediately to 0 and the process starts repeating. What is the fraction of electrons missing the time focus at the probe D?

4. Coefficient of friction (12 pts)

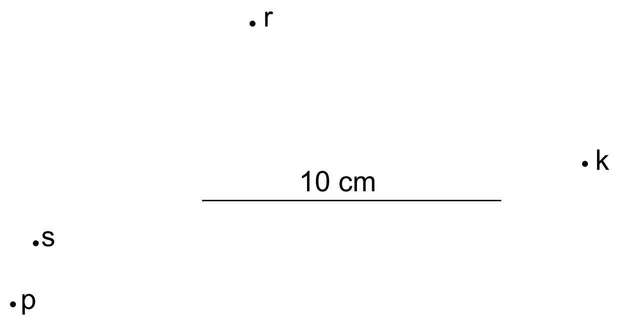
Equipment: a wooden brick, a spherical ball, board and ruler (the mass ration of the brick and ball is provided).
1) Determine the static coe cient of friction between the board and the brick.
2) Determine the static coe cient of friction between the ball and the brick.

5. Rotating disk (7 pts)

A lamp is attached to the edge of a disk, which moves (slides) rotating on ice. The lamp emits light pulses: the duration of each pulse is negligible, the interval between two pulses is $\tau = 100$ ms. The first pulse is of orange light, the next one is blue, followed by red, green, yellow, and again orange (the process starts repeating periodically). The motion of the disk is photographed using so long exposure time that exactly four pulses are recorded on the photo (see figure). Due to the shortness of the pulses and small size of the lamp, each pulse corresponds to a colored dot on the photo. The colors of the dots are provided with lettering: O— orange, S — blue, P —red, R — green, and K — yellow). The friction forces acting on the disk can be neglected.

1) Mark on the figure by numbers (1–4) the order of the pulses (dots). Motivate your answer. What can be said about the value of the exposure time?

2) Using the provided figure, find the radius of the disk R, the velocity of the center of the disk v and the angular velocity ω (it is known that $\omega < 60$ rad/s). The scale of the figure is provided by the image of a line of length $l = 10$ cm;



6. Truck (7 pts)

1) A rope is put over a pole so that the plane of the rope is perpendicular to the axes of the pole, and the length of that segment of the rope, which touches the pole is l, much shorter than the radius of the pole R, see figure (a). To the one end of the rope, a force T is applied; the sliding of the rope can be prevented by applying a force T_1 to the other end of the rope. Express the ratio T_1/T via l, R, and μ, where μ is the coe cient of friction between the rope and the pole.

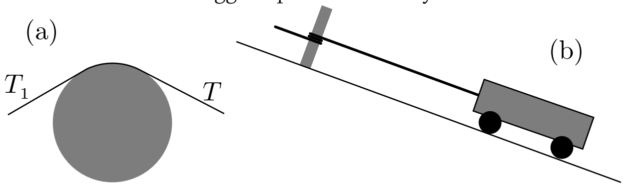
2) Answer the first question, if l is not small (i.e. without the assumption $l \ll R$).

Remark: you may use equality

$$\lim_{x\to 0}(1+nx)^{1/x} = e^n.$$

3) The rope makes exactly $n = 2$ winds around the pole. One end of the rope is attached to a truck standing on a slope (slanting angle $\phi = 10°$); the mass of the truck $m = 20$ t, see figure (b). Find the force F, needed to apply to the other end of the rope, in order to keep the truck at rest. Use the numerical value $\mu = 0{,}3$. All the other friction forces acting upon the truck can be neglected.

4) How does the answer change, if the cross-section of the pole is not circular, but instead, egg-shaped? Motivate your answer.



7. To the Mars (10 pts)

In this problem, we study a project for flying to Mars. At the first stage, the space ship switches on the rocket engines and obtains an initial velocity v_0. You may assume that during the first stage, the height of the space ship (from the surface of the Earth) remains much smaller than the radius of the Earth $R_0 = 6400$ km. At the second stage, the space ship performs a ballistic motion in the gravity field of the Earth: upon achieving a height, which is much larger than the radius of the Earth R_0, but much smaller than the orbital radius of the Earth, $R_e = 1.5 \cdot 10^8$ km.

1) Find the relationship between the residual velocity v_1 (with respect to the Earth) at the end of the second stage, and the quantities v_0, R_0, and the free fall acceleration at the surface of the Earth g (for the subsequent questions you may use the numeric value $g \approx 9.8\,\text{m/s}^2$).

At the third stage, the space ship performs a ballistic motion in the gravity field of the Sun, up to reaching an immediate neighborhood of the Mars. The trajectory is chosen by minimizing the residual launch velocity v_1 (required for achieving the Mars).

2) Sketch the trajectory.

3) Find the flight time T. You may use the following numeric data: the orbital velocity of the Earth $v_e = 30$ km/s, orbital radius of the Mars $R_m = 2.3 \cdot 10^8$ km.

4) Find the previously considered launch velocity v_0 and the terminal (i.e. at the end of the third stage) velocity v_t of the space ship with respect to the Mars.

5) The required mass of the fuel M can be found from the formula $v = u \ln[(M+m)/m]$, where u is the speed of the gas at the outlet of the engine (with respect to the space ship), and m is the useful mass of the ship (when all the fuel is exhausted). You may assume $m \ll M$ and use the numerical value $u = 1$ km/s. Find, how much more fuel is needed for the Mars flight, as compared to simply escaping the Earth gravity field, if the useful space ship mass is equal in both cases.

8. Laser (12 pts)

Apparatus: Laser (wavelength $\lambda = 650$ nm), ruler, stand, a strip of reflecting material, a sheet of paper with a circular hole (in your pack of paper sheets), pencil. Note that the reflecting material provided to you is coated with a layer of densely packed tiny glass spheres of equal diameter.

1) Describe the position and geometry of the di raction pattern, which can be observed, when the laser beam falls onto the strip; use di erent incidence angles.

2) Provide a qualitative (approximate) explanation for the observed phenomenon.

3) Estimate the diameter of these glass spheres.