1. Rubber fiber (12 pts)

Fibers made of elastic rubber can be stretched to lengths l, much longer than the length in undeformed state l_0. For such rubbers, the net volume of the fiber remains constant.

1) Express the cross-sectional area S of such a fiber in a deformed state via its length l and initial dimensions l_0, S_0 (1 pt).

2) For small deformations of an elastic material, the stretching force F and deformation x are related to each other by the Hooke's law $F = k_0 x$, where the sti ness $k_0 = E_0 S_0 / l_0$ and E_0 is the Young's modulus of the rubber. For non-small (possibly large, $l \gg l_0$) deformations of elastic rubber, however, the Hooke's law is substituted by a non-linear law, $F(l) = a + \frac{b}{l}$ (breaking of this law at very large values of l will not be studied here). Express the constants a and b in terms of l_0, S_0, and E_0 (2 pt).

3) Suppose such a fiber is stretched by some force up to the length l. A small change ΔF of the stretching force results in a small change in the length $\Delta l \ll l$. Express ΔF in terms of l, l_0, S_0, E_0, and Δl (1 pt).

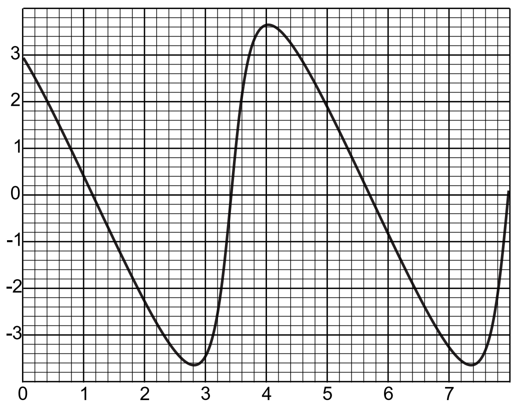
4) Suppose a small body is fixed to an one end of the fiber and the system is put into rotation around the other end of the fiber. In the case of a circular motion of the body, express the length of the fiber l via l_0, S_0, E_0, and the kinetic energy of the body K (the kinetic energy of the fiber and gravity can be neglected). (1.5 pts)

5) Let us analyse a slightly non-circular motion of the body. Let us describe the motion of the system by the length change of the fiber $r(t) = l(t) - l(0)$, the radial $v_r(t)$ and tangential $v_t(t)$ velocities of the body (the components respectively parallel and perpendicular to the fiber). The initial values of these quantities are designated as $L \equiv l(0)$, $V_r \equiv v_r(0)$, and $V_t \equiv v_t(0)$. The values L and V_t are chosen so that if the initial radial velocity were zero, the motion would be circular. Write down two independent equations relating $r(t)$, $v_r(t)$, and $v_t(t)$ to each other (using also the mass of the body m, together with the parameters L, V_r, V_t, l_0, S_0, E_0). (3.5 pts).

6) Find the relationship between $r(t)$ and $v_r(t)$ (containing also the parameters m, L, V_r, V_t, l_0, S_0, E_0) assuming that $|r| \ll L$, and find the period T of small oscillations of $r(t)$. Simplify the expression of T for $L \gg l_0$ (3 pts).

2. Planets (6 pts)

Two planets move along circular orbits around a star of mass $M = 2.0 \cdot 10^{30}$ kg; gravitational constant $G = 6.67 \cdot 10^{-11}\ \mathrm{m^3/kg \cdot s^2}$. The dependence of the angular distance between a planet and the star on time, as seen from the other planet, is depicted in figure.



1) What is the ratio of the radii of the planets k (2 pts)?

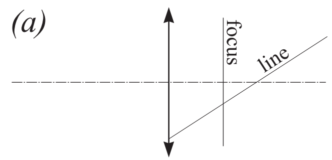
2) Determine the value of the unit on the vertical axis (or express it in terms of k, if you were unable to find it) (2 pts).

3) What are the orbital radii of the planets, if the unit on the horizontal axis equals to one year (2 pts)?

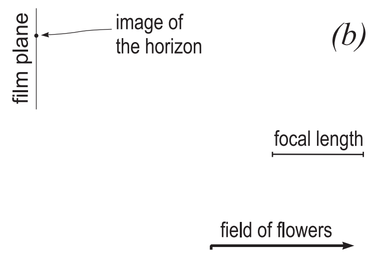
3. Tilt-shift lens (6 pts)

1) Show that an image of a straight line created by a thin lens is also a straight line. Consider two-dimensional geometry only, i.e. assume that the main optical axis and the straight line lay in the same (x, y) surface. Hint: use the coordinate system, where the origin coincides with the center of the lens and represent lines algebraically, e.g. $y = ax + b$. Make use of the formula of thin lens $f^{-1} = x^{-1} - x'^{-1}$ ($x > 0$ and x' are the x-coordinates of a point and its image, respectively) (2 pts).

2) In figure *(a)*, draw the image of the given line and indicate, which parts of the image are virtual, and which are real (2 pts).

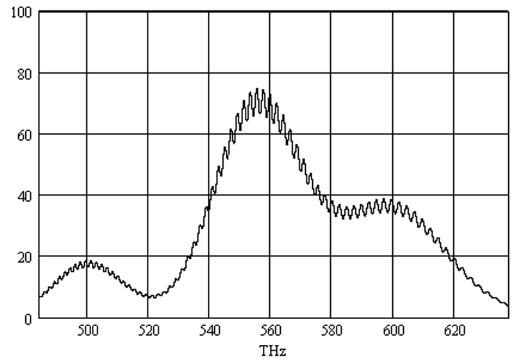


3) Photographer wants to take a photo of a field of flowers. In order to get image where all the flowers (both the close and far ones) are sharp, he has to use a lens with tilt-shift (TS) capabilities (either an ordinary camera with TS lens, or a large-format camera, where the entire lens compartment can be freely positioned). The field of flowers (which extends e ectively to infinity) and the image of its distant edge, together with the image plane are depicted in figure *(b)*. Reconstruct the position of the lens, the focal length of which is provided as a scale (2 pts).



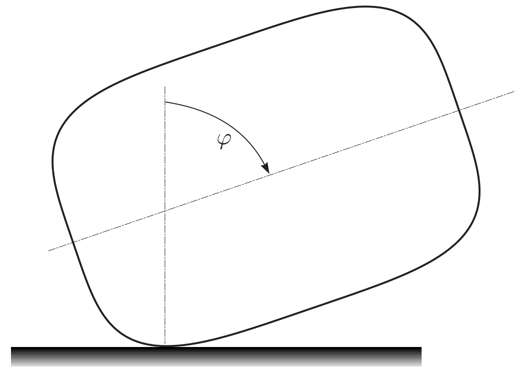
4. Transparent film (6 pts)

A thick glass plate is coated by a thin transparent film. The transmission spectrum of the system is depicted in graph (light falls normal to the plate). The refractive index of the film $n \approx 1.3$. What is the thickness of the film d?



5. 4th order ellipse (6 pts)

4th order ellipse is defined by equation $\frac{x^4}{a^4} + \frac{y^4}{b^4} = 1$, where a and b are the lengths of the half- axes. Consider an homogeneous cylinder, the cross-section of which is 4th order ellipse. The position of the cylinder is measured by the angle $0 \le \varphi \le \pi/2$ between the vertical direction and a longer half-axes, see figure.



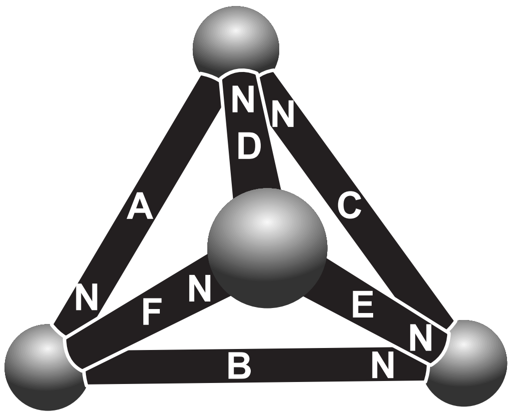
1) What are the equilibrium positions of the cylinder laying on an horizontal surface (3.5 pts)?

2) Sketch on graph the net torque of gravity and surface reaction forces with respect to the contact point of the cylinder and surface as a function of φ ($0 \le \varphi \le \pi/2$). For the axis of torque, you do not need to indicate any quantitative scale (1.3 pts).

3) Which equilibrium positions are stable and which are not? Motivate your answer (1.2 pts).

6. Magnets (6 pts)

Certain type of magnetic toys are made up of ferromagnetic spheres and permanent magnets of cylindrical shape. These building blocks can be used to build, for instance, a tetrahedron, see figure (letter “N” marks the northern end of a magnet). Assume that all these permanent magnets are identical and each of them alone can create a magnetic flux Φ (assuming the both ends of the magnet are in contact with a U-shaped large piece of ferromagnetic material, so that a closed ferromagnetic contour is formed). Assume also that due to high magnetic permeability of the material of the building blocks, all the magnetic field lines are constrained inside of them (i.e. in the surrounding medium, the magnetic inductance $B = 0$).



1) Let us designated the fluxes in each permanent magnet (magnets A–F in figure) by Φ_A – Φ_F. Write down equation relating Φ_A, Φ_B, and Φ_C to each other (and possibly to Φ) (1 pt).

2) Write down equation relating Φ_A, Φ_B, and Φ_F to each other (and possibly to Φ) (1 pt).

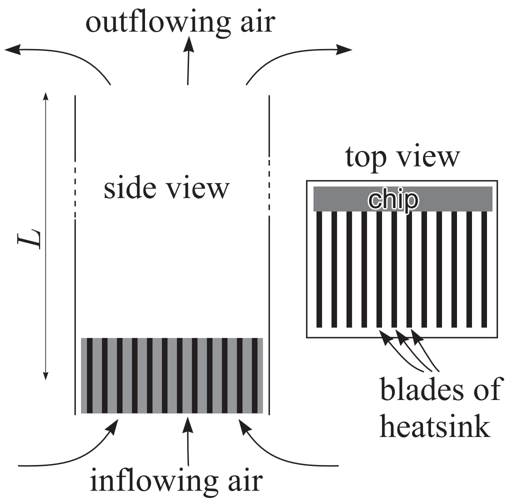
3) Find the ratio Φ_F/Φ_C (1 pt).

4) Find the magnetic fluxes in each permanent magnet (2 pts).

5) Which of the magnets is the most di cult one to remove? Motivate your answer (1 pts).

7. Passive air-cooling (9 pts)

Consider a passive cooling system depicted in figure. Cold air (at normal conditions: $p_0 = 10^5\,\mathrm{Pa}$, $T_0 = 293\,\mathrm{K}$) flows over the heat sink of a chip of power dissipation $P = 100\,\mathrm{W}$, into a vertical pipe of length $L = 1\,\mathrm{m}$ and cross-sectional area $S = 25\,\mathrm{cm}^2$. After passing the pipe, air enters the ambient room. Assume that the air inside the pipe becomes well mixed; neglect the viscous and turbulent friction of air inside the pipe and heat sink. Air can be considered as an ideal gas with adiabatic exponent $\gamma = 1.4$ and molar mass $\mu = 29\,\mathrm{g/mol}$.



1) Express heat capacitance at constant pressure c_p via quantites γ and R (1 pt).

2) Find a relationship between the outflowing air density ρ and temperature T (the relationship may contain also the parameters defined above) (2 pts).

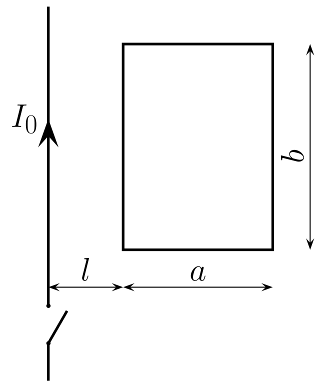
3) Find a relationship between the air flow velocity in the pipe v and outflowing air density ρ (the relationship may contain also the parameters defined above) (2 pts).

4) Express the power disspation P in terms of the air flow velocity v, the outflowing air temperature T, and density ρ (the relationship may contain also the parameters defined above) (2 pts).

5) What is the temperature T of the outflowing air? In your calculations, you may use approximation $T - T_0 \ll T_0$(2 pts)?

8. Loop of wire (7 pts)

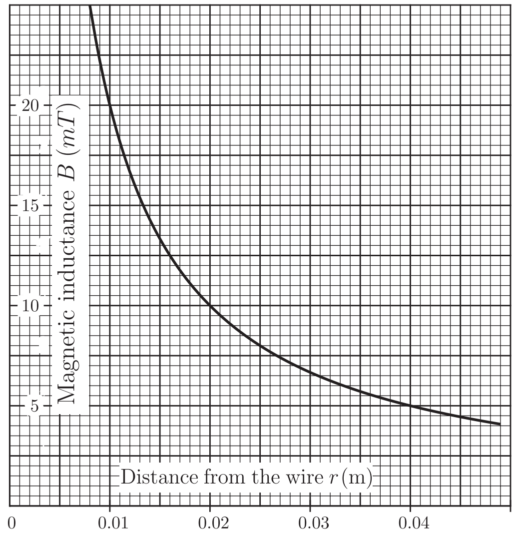
Consider a rectangular loop of wire with dimensions $a = 0.03\,\mathrm{m}$ and $b = 1.0\,\mathrm{m}$, one side of which is parallel to another long straight wire carrying current $I_0 = 1000\,\mathrm{A}$, at distance $l = 0,01\,\mathrm{m}$, see figure. The magnetic inductance of such current is plotted as a function of the distance from the wire in attached graph. The Ohmic resistance of the loop is $R = 1,0\,\Omega$, the inductance is negligible.



1) Calculate the magnetic flux Φ through the loop (2 pts).

2) At a certain moment of time, the current in the long wire is switched o . What is the net charge Q flowing through a fixed cross-section of the wire of the loop (3 pts)?

3) What is the net momentum p given to the loop during the switch-o of the current (express it in terms of Q and the given quantities, if you were unable to calculate Q) (2 pts)?



9. Experiment (15 pts)

The black box contains a nonlinear element (active resistance) and a capacitor, connected sequentially. Find the capacitance C of the capacitor (5 pts) and the $V - I$ characteristic of the nonlinear element (6 pts). Note that *(a)* the electrolytic capacitor accepts only one polarity of charge (indicated by the colors of the output wires of the black box); *(b)* the $V - I$ characteristic cannot be expected to be symmetric with respect to $I = 0$. However, you are requested to study the range $I > 0$ corresponding to the discharge of the capacitor. Tabulate your measurement data and draw appropriate graphs (4 pts). Experimental equipment: batteries, wires, multimeter, stopwatch, graphic paper.