Tallinn 2003

Complete solution of each subquestion gives two points. You have five hours for solving.

1. Volleyball (8 points)

Consider a simplified model of a volleyball: a thin spherical envelope filled with air. The envelope material is non-stretchable, easily foldable. The excess pressure inside the ball $\Delta p = 20\,\text{kPa}$, the ball radius $R = 10\,\text{cm}$ and mass $m = 400\,\text{g}$ (the air mass inside the ball is negligible). You can neglect the dependence of the excess pressure on the deformation of the ball.

1) The ball is pressed between two parallel rigid plates, the distance between of which is $2R - 2h$ (so that the height of the deformed segments is $h = 1\,\text{cm}$). Find the force between the ball and a plate N.

2) Ball moves with velocity $v_0 = 2\,\text{m/s}$ and hits a rigid wall. Find the maximal height of the deformed segment h_m ...

3) ...and the collision time τ.

4) For small excess pressures, the ball can lose its spherical shape even in those points, which are not in touch with the wall. Which condition between the quantities Δp, R, m and h has to be satisfied in order to ensure that such a loss of sphericity is negligible?

2. Heat flux (4 points)

Heat resistivity is equal to the ratio of the temperature difference between the end-points of a wire of unit cross-section and unit length, and the heat flux (unit: W) through this wire.

1) Microprocessor of power $P = 90\,\text{W}$ has a water-cooling system. The chip and flowing water are separated by a copper plate of thickness $d = 5\,\text{mm}$ and cross-section area $s = 100\,\text{mm}^2$. What is the temperature difference between the processor and water? The copper heat resistivity is $\rho = 2{,}6\,\text{mm·K/W}$.

2) A wire is made of different alloys, its heat resistivity ρ as a function of the coordinate along the wire is given in the attached graph. The cross-section area of the wire is $S = 1\,\text{mm}^2$, its length $l = 4\,\text{cm}$. Find the heat flux through the wire, if one end of the wire is kept at the temperature $100°\text{C}$, and the other end — at $0°\text{C}$.
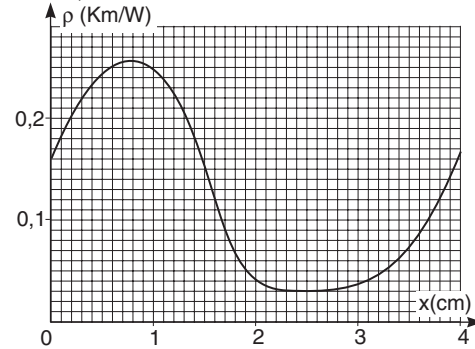

3. Gravitation (6 points)

1) Find the free falling acceleration g_0 at the surface of such a spherical planet, which has mass M and material density ρ (in what follows, M and ρ are assumed to be constant).

2) Is it possible that at the surface of a non-spherical planet, there is a point with free falling acceleration $g > g_0$? Motivate your answer.

3) For which planet shape the maximum of the free falling acceleration is achieved? Answer can be given in polar coordinates, the expression can contain one unspecified constant.

4. Tunnel diode (8 points)

Tunnel diode is a semi-conductor device, similar to the ordinary diode, the volt-amper characteristic of which is given in the attached graph. The circuit below describes a simple amplifier. The resistance $R = 10\,\Omega$, battery voltage $\mathcal{E} = 0{,}25\,\text{V}$.
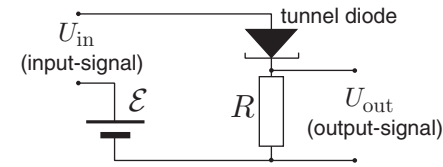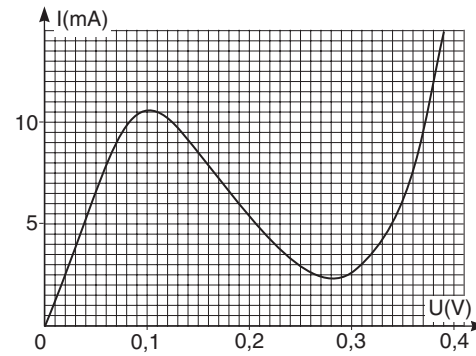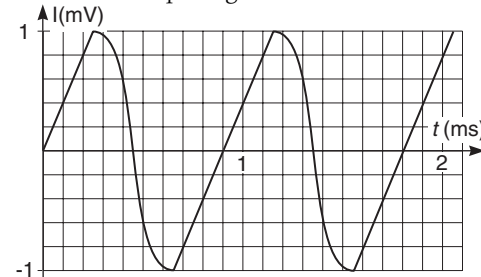

1) Find the current in the circuit, if $U_{in} + \mathcal{E} = 0{,}08\,\text{V}$.

2) Find the output voltage $U_{out\,0}$ if $U_{in} = 0\,\text{V}$.

3) Find the output signal $U_{out} - U_{out\,0}$ if $U_{in} = 1\,\text{mV}$.

4) The input signal is given in the graph below. Sketch the output signal as a function of time.

5. Vibration (10 points)

Consider a smooth horizontal surface, which is moved periodically back and forth along the horizontal x-axis: during the first semi-period τ, the surface velocity is u, during the second semi-period — $-u$. A brick of mass m is put on that surface; the friction coefficient between the surface and brick is μ, free falling acceleration is g.

1) The brick has initial x-directional velocity v. Which condition between the quantities g, μ, v, and τ has to be satisfied in order to ensure that the brick velocity change during a semi-period is negligible? Further we assume that this condition is satisfied.

2) The brick is kept in motion along x-axis by a force F_x in such a way that the mean brick velocity is v. Sketch graphically the dependance $F_x(v)$.

3) The brick is kept in motion along (horizontal) y-axis by a force F_y in such a way that the mean brick velocity is v. Find the dependance $F_y(v)$.

4) Until now we have ignored the dependence of the friction coefficient on the sliding velocity. Further let us assume this dependance is given by the graph below. The brick is kept in motion along x-axis by a force F_x in such a way that the mean brick velocity is v. Sketch graphically the dependance $F_x(v)$ taking $u = \frac{3}{4}w_0$.
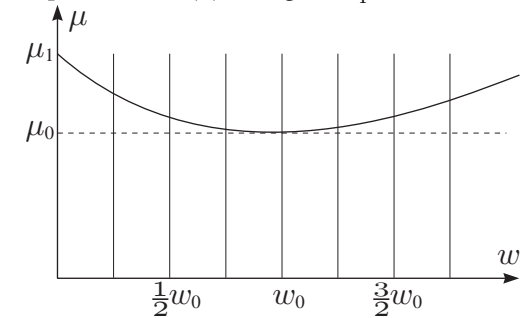
5) The brick is put on the surface, there are no external forces. What is the brick's terminal velocity v? Provide the answer as a function of u.

6. Charged particle (12 points)

A particle of mass m and charge q is in a homogeneous magnetic field with induction B (the vector is parallel to the z-axis). The characteristic time of the system is the cyclotron period of the particle $T_B = 2\pi m / Bq$. The system is situated in between two parallel electrodes, which can be used to create an homogeneous, parallel to the x-axis electric field E.

1) The particle is at rest. At the moment of time $t = 0$, the electric field E is switched on; after a short time interval τ ($\tau \ll T_B$), it is switched off, again. What will be the trajectory of the particle?

2) Let p_x and p_y denote the x- and y components of the momentum of the particle. Sketch the trajectory of the particle in (p_x, p_y)-plane and depict the vectors of the momentum for the moments of time $t_n = nT_B/4$ ($n = 1, 2, 3$ and 4).

3) Consider the situation when the on-off switching of the electric field is done periodically, starting with $t = 0$, after equal time intervals $\Delta t = T_B/4$. Sketch the particle trajectories in (p_x, p_y)- and (x, y)-planes.

4) Let the period be short, $\Delta t \ll T_B$ (but still much longer than the duration of pulse, $\Delta t \gg \tau$). Show that after the n-th pulse (at the time moment $t_n = n\Delta t$), the momentum of the particle can be represented as the sum of n vectors $\vec{p}_i$, where all the component-vectors are equal in modulus (the modulus being independent of n), and the neighboring vectors ($\vec{p}_i$ and $\vec{p}_{i+1}$, $i = 1, 2, \ldots$) have equal angles between them.

5) Consider the limit case $\Delta t \to 0$, so that $E\tau/\Delta t \to E_k$ (E_k denotes the time-average of the elctric field). Sketch the particles trajectory in (p_x, p_y)-plane and express the particles mean velocity (vectorially; averaged over the cyclotron period) via the quantities E_k and B.

6) Let us return to the non-zero (but still small, $\Delta t \ll T_B$) time-intervals. Let us consider the case when the pulses are of variable polarity: for the $2n$-th pulse, the electric field is $+E$, and for the $2n+1$st pulse $-E$. Find the particles average velocity, (vectorially) averaged over the cyclotron period.

7. Telescope (12 points)

As it is well known, a telescope makes it possible to see the stars in daylight. Let us study the problem in more details. Consider a simplified model of the eye: a single lens with focal length $f = 4\,\text{cm}$ and diameter $d = 3\,\text{mm}$ creating an image on screen (retina). The model of a telescope is similar: a lens of focal length $F = 2\,\text{m}$ and diameter $D = 20\,\text{cm}$ creating an image in focal plane (where eg. a film can be put). In your calculations, the following quantities can be used: the density of the light energy radiated from a unit Solar surface in unit time w_0 (the light power surface density); the ratio of the star and Sun distances $q = 4 \cdot 10^5$ (we assume that the star is identical to the Sun); Solar angular diameter $\phi \approx 9\,\text{mrad}$. *Remark:* If the answer contains w_0 then numerical answer is not required.

1) Consider a sheet of paper, the normal of which is directed towards the Sun. What is the surface density of the light power w_1 arriving to the sheet from the Sun?

2) Find the net power P_2 of the light, which is focused by the telescope into the image of the star.

3) Assume that blue sky is as bright as a sheet of gray paper illuminated by Sun. You may assume that in the direction, perpendicular to the sheet, the ratio of the light power scattered by the paper into a 1-steradian space angle, to the net light power arriving to the sheet, is $\alpha \approx 0.1$ (this corresponds to the dissipation of ca 70 % light energy in the gray paper). What is the surface density of the light power in the focal plane of the telescope w_3, due to the blue sky?

4) While studying the star image, let us ignore all the effects other than diffraction. Estimate the surface density of the light power in the center of the star image w_2 (in the focal plane of the telescope), due to the light arriving from the star.

5) Provide an expression for the ratio of the surface densities of the light powers k in the middle of the star image, and in a point farther away from it.

6) Is it possible to see a star in daylight using a telescope? Plain eye? Motivate yourself.

8. Experiment (12 points)

Determination of attraction force between iron plate and a permanent magnet as a function of distance. Tools: iron plate, wooden brick, ruler, dynamometer, paper stripes.

Attention! the permanent magnets are very strong, keep them far from credit cards etc. Avoid also hitting them against each other and against the iron plate, because they are fragile and can be broken.

1) Determine the static and dynamic coefficients of friction between the iron plate and a paper stripe. Draw the scheme of your set-up.

2) (4 points) Determine the attraction force between the iron plate and a magnet for those distances which allow direct usage of dynamometer. Draw the scheme of your set-up.

3) (4 points) Determine the attraction force between the iron plate and a magnet for smaller distances. For that purpose, you can use the wooden brick sliding down an inclined plate and hitting the magnet. You do not need to study the zero-distance (direct contact of magnet and iron plate) case. Draw the scheme of your set-up. Depict all the measurement result graphically.

4) Join two permanent magnets by a bridge made of a piece of iron (a) as shown in figure. Put a stripe of paper (b) on the iron plate (c) and put the system of magnets upon it. Determine the attraction force between the system of magnets and iron plate.